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DIFFUSION OF CHARGED PLASMA PARTICLES IN A MAGNETIC FIELD

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In the last few years, a large number of theoretical and experimental studies have been published which have been devoted to the investigation of the diffusion of charged plasma particles in a magnetic field. Interest in these studies has been stimulated primarily by the extensive development of research on the confinement and heating of plasma in a magnetic field conducted in connection with the problem of controlled thermonuclear reactions (see, for example, reference [1-3]) and with astrophysical investigations of plasma [4-5].

A number of books dealing with plasma physics [1-3, 6-12] discuss certain aspects of the theory of diffusion in a magnetic field and cite individual experimental results. However, there is no systematic survey of the present status of diffusion research. The present paper is aimed at filling this gap: it sets forth the theory of diffusion of a stable plasma and offers a survey of the principal experimental investigations of diffusion in a magnetic field.*

I. THEORY OF DIFFUSION DUE TO PARTICLE COLLISIONS

The effect of a magnetic field on the diffusion of charged particles in a gas and their motion in an electric field were noted in the early work of Townsend [14]. In these and many subsequent studies [14-15], the directed motion of charged particles in a neutral gas was treated by the approximate free-path method. A more detailed analysis of the processes

*It should be noted that various plasma instabilities have been detected in many experiments. The problems of the theory of stability are not considered in this survey (see, for example, reference [13]).

of transport of charged particles in a neutral gas, based on the solution of the kinetic equation and averaged equations of motion, was given in ref. [16-21]. Results of these investigations were used in the treatment of dipolar diffusion in a weakly ionized gas [22, 20, 21].

A large number of studies have been devoted to the processes of transport across a magnetic field in a completely ionized gas. The transport phenomena were investigated by means of averaged equations of motion for charged particles [23-25, 6], by means of the kinetic equation [26-34], and by methods based on the treatment of the displacement of individual particles [35-39]. The collective Coulombic interactions of charged particles were treated in the majority of cases as a set of independently pair collisions with a maximum interaction radius equal to the Debye radius. A special investigation made with the use of quantum field theory methods showed the permissibility of such treatment in the analysis of transport processes [40, 41].

A number of papers [42-45] discuss diffusion across a magnetic field in a strongly ionized gas for conditions under which collisions of charged particles with neutral particles and with one another are significant.

We shall not dwell on the various methods employed in the theoretical studies of transport phenomena in a magnetic field. In order to obtain the general relations determining the rate of diffusion under the influence of concentration gradients in arbitrary magnetic fields, we shall use the simplest method one which is based on the solution of approximate equations of the motion of charged particles. Transverse diffusion in strong magnetic fields will then be considered in detail, and expressions for diffusion flows will be obtained from an analysis of particle displacements produced by the collisions. In order to avoid encumbering the presentation, the diffusion caused by temperature gradients (thermal diffusion) will not be considered.

1. Theory of Diffusion Based on the Equations of Particle Motion

1. Averaged equations of motion. The averaged equation of motion of charged particles may be written as follows:

$$m_\alpha \left[\frac{\partial \mathbf{u}_\alpha}{\partial t} + (\mathbf{u}_\alpha \nabla)\, \mathbf{u}_\alpha \right] = Z_\alpha e \mathbf{E} + Z_\alpha e \frac{[\mathbf{u}_\alpha \mathbf{H}]}{c} - \frac{\nabla p_\alpha}{n_\alpha} + m_\alpha \frac{\delta \mathbf{u}_\alpha}{\delta t}. \tag{1.1}$$

Here $Z_\alpha e$ is the charge; m_α, the mass; u_α, the vector of the average (directed) velocity; n_α, the concentration; p_α, the pressure; $m_\alpha \frac{\delta \mathbf{u}_\alpha}{\delta t}$ is the momentum change resulting from the collisions (the "friction force") (all the quantities pertain to particles of the "α" kind)*; E and H are the strengths of the electric and magnetic fields. The pressure p_α for a Maxwellian distribution of random particle velocities is given by the equality

$$p_\alpha = n_\alpha T_\alpha \tag{1.2}$$

(T_α is the temperature in energy units).

As we know, equation (1.1) is a result of the kinetic equation (see reference [9]). In the general case, the last term, $m_\alpha \frac{\delta \mathbf{u}_\alpha}{\delta t}$, can be determined only by integrating the kinetic equation. The quantity $m_\alpha \frac{\delta \mathbf{u}_\alpha}{\delta t}$ may depend not only on the concentration and temperature of the various particles, but also on the magnetic field. In certain approximation, however, the magnitude of the friction force may be represented by the relation [6, 23, 24].

$$m_\alpha \frac{\delta \mathbf{u}_\alpha}{\delta t} = -m_\alpha \sum_{(\beta)} \nu_{\alpha\beta} (\mathbf{u}_\alpha - \mathbf{u}_\beta). \tag{1.3}$$

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*We shall designate the quantities pertaining to electrons by the subscript $\alpha = e$; those pertaining to ions, by the subscript $\alpha = i$; and those pertaining to neutral atoms, by the subscript $\alpha = n$.

Each of the terms on the right side of the equation gives the averaged change in the momentum of a particle of the "α" kind per unit time, resulting from its collisions with particles of the "β" kind. It is natural to assume that the "friction force" determined by the collisions of particles of the "α" and "β" kind is proportional to their relative velocity. The quantity $v_{\alpha\beta}$ is a certain effective frequency of particle collisions. Since the momentum is conserved in the collisions, the quantities $v_{\alpha\beta}$ and $v_{\beta\alpha}$ should be related by the expression

$$n_\alpha m_\alpha \nu_{\alpha\beta} = n_\beta m_\beta \nu_{\beta\alpha}. \tag{1.4}$$

We shall subsequently consider stationary or quasi-stationary processes in which

$$\left|\frac{\partial \mathbf{u}_\alpha}{\partial t}\right| \ll \left|\frac{\delta \mathbf{u}_\alpha}{\delta t}\right| \tag{1.5}$$

and the first term on the left side of the transport equation may be ignored. Further, we shall consider the perturbations (concentration gradients and the electric field) to be small, so that the quadratic terms $(\mathbf{u}_\alpha\nabla)\mathbf{u}_\alpha$ on the left side of equation (1.1) may also be ignored. They may be ignored when the directed velocity of particles is much smaller than the random (thermal) velocity:

$$\mathbf{u}_\alpha \ll \sqrt{\frac{T_\alpha}{m_\alpha}}. \tag{1.6}$$

This inequality is obviously a condition of the diffusional character of the transport phenomena.

Keeping the above simplifications in mind and using equalities (1.2) and (1.3), we may write equation (1.1) as an equation of equilibrium between the electric force, the Lorenz force, the pressure gradient (per particle) and the effective friction force

$$Z_\alpha e\mathbf{E} + Z_\alpha e\frac{[\mathbf{u}_\alpha \mathbf{H}]}{c} - T_\alpha \frac{\nabla n_\alpha}{n_\alpha} - m_\alpha \sum_{(\beta)} \nu_{\alpha\beta}(\mathbf{u}_\alpha - \mathbf{u}_\beta) = 0. \tag{1.7}$$

In equation (1.7) the temperature is assumed to be constant and has been put outside the gradient symbol, since the thermal diffusion will not be considered.

System of equations (1.7) permits determination of the directed velocities of all the plasma particles, i.e., makes it possible to solve the problem of the directed motion of particles.

2. Directed motion of charged particles in a neutral gas. Let us first consider the motion of the charged particles of a weakly ionized gas in which collisions of the charged particles with one another are not essential, i.e., the frequencies of collisions of electrons and ions with neutral atoms are much greater than their collisions with one another:

$$\nu_{en} \gg \nu_{ei}, \quad \nu_{in} \gg \nu_{ie}. \tag{1.8}$$

In this case, transport equations (1.7) for charged particles of various types are independent and are of the form

$$Z_\alpha e\mathbf{E} + Z_\alpha e\frac{[\mathbf{u}_\alpha \mathbf{H}]}{c} - T_\alpha \frac{\nabla n_\alpha}{n_\alpha} - m_\alpha \nu_{\alpha n}\mathbf{u}_\alpha = 0. \tag{1.9}$$

Here $\nu_{\alpha n}$ is the frequency of collisions of particles of the "α" kind with neutral atoms; we have allowed for the fact that when the degree of ionization is small, the directed velocity of the neutral atoms is much less than the directed velocity of the charged particles.

If we project vector equation (1.9) on the direction of the magnetic field, we find an expression for the longitudinal directed velocity

$$\left.\begin{aligned} u_{\alpha\|} &= -D_{\alpha\|}\frac{\nabla_\| n_\alpha}{n_\alpha} + \mu_{\alpha\|}E_\|, \\ D_{\alpha\|} &= \frac{T_\alpha}{m_\alpha \nu_{\alpha n}}, \quad \mu_{\alpha\|} = \frac{Z_\alpha e}{m_\alpha \nu_{\alpha n}} \end{aligned}\right\} \tag{1.10}$$

($u_{\alpha\|}$, $E_\|$, $\nabla_\| n_\alpha$ are the projections of the corresponding vectors on the direction of the magnetic field).

The projection of equation (1.9) on a plane perpendicular to the magnetic field makes it possible to determine the transverse components of the directed velocity, i.e., the transverse velocity component in the direction of the concentration gradient and of the electric field, $u_\perp$, and the velocity component perpendicular to the magnetic field and concentration gradient, $u_\top$:

$$\left.\begin{aligned} \mathbf{u}_{\alpha\perp} &= -D_{\alpha\perp}\frac{\nabla_\perp n_\alpha}{n_\alpha} + \mu_{\alpha\perp}\mathbf{E}_\perp, \\ D_{\alpha\perp} &= \frac{T_\alpha}{m_\alpha \nu_{\alpha n}\left(1+\frac{\omega_\alpha^2}{\nu_{\alpha n}^2}\right)}, \quad \mu_{\alpha\perp} = \frac{Z_\alpha e}{m_\alpha \nu_{\alpha n}\left(1+\frac{\omega_\alpha^2}{\nu_{\alpha n}^2}\right)}; \end{aligned}\right\} \tag{1.11}$$

$$\mathbf{u}_{\alpha\top} = \frac{cT_\alpha [\mathbf{h}\nabla n_\alpha]}{Z_\alpha e H n_\alpha\left(1+\frac{\nu_{\alpha n}^2}{\omega_\alpha^2}\right)} - \frac{c[\mathbf{hE}]}{H\left(1+\frac{\nu_{\alpha n}^2}{\omega_\alpha^2}\right)}. \tag{1.12}$$

Here $\mathbf{E}_\perp, \nabla_\perp n$ are the projections of the vectors E and ∇n onto the plane perpendicular to the magnetic field; h is the unit vector in the direction of the magnetic field; ω_α is the Larmor frequency

$$\omega_\alpha = \frac{|Z_\alpha| eH}{m_\alpha c}. \tag{1.13}$$

In order to describe the motion of particles in the direction of the concentration gradient and of the electric field, we have introduced the coefficients of transverse and longitudinal diffusion, $D_{\alpha\|}, D_{\alpha\perp}$, and the corresponding mobilities $\mu_{\alpha\|}, \mu_{\alpha\perp}$. The relation between them is given by Einstein's relations

$$\frac{D_{\alpha\|}}{\mu_{\alpha\|}} = \frac{D_{\alpha\perp}}{\mu_{\alpha\perp}} = \frac{T_\alpha}{Z_\alpha e}. \tag{1.14}$$

The difference between the quantities $D_{\alpha\|}$ and $D_{\alpha\perp}$ (or $\mu_{\alpha\|}$ and $\mu_{\alpha\perp}$) determines the anisotropy of the transport processes in the presence of a magnetic field.

Let us note that the motion in a direction perpendicular to the concentration gradient and the electric field with large magnetic fields (when $\omega_\alpha \gg \nu_{\alpha n}$) is determined by the drift velocity of the charged particles.

Using the equations of motion, we have obtained approximate expressions for the flow of charged particles in a neutral gas. Let us compare these expressions with more precise formulas obtained for cetrain cases with the aid of the kinetic equation.

Integration of the kinetic equation can be performed if one examines the motions of electrons in a gas under conditions where only elastic electronic collisions are important. The examination leads to the following expressions for the diffusion coefficients [20]:

$$De_{\|} = \frac{1}{3}\int\limits_{(v)} \frac{v^2}{\nu_{en}} f_e(v)\,d\mathrm{v}, \quad De_{\perp} = \frac{1}{3}\int\limits_{(v)} \frac{v^2 \nu_{en}}{\omega_e^2 + \nu_{en}^2} f_e(v)\,d\mathrm{v}, \tag{1.15}$$

where v is the electron velocity, $f_e(v)$ is the velocity distribution function

$$\nu_{en}^* = n_n v s^*_{en}(v), \; s_{en}^* = \int\limits_{(\Omega)} \sigma_{en}(v, \vartheta)(1 - \cos\vartheta)\,d\Omega, \tag{1.16}$$

v^*_{en} is the "diffusion" frequency of collisions s^*_{en} is the cross section of momentum transfer, $\sigma_{en}(v, \vartheta)$ is the differential cross section of the scattering of electrons by atoms; the integration of (1.15) is performed over the entire volume in the space of electron velocities.

Expressions (1.15) and (1.10)--(1.11) are identical if the diffusional frequency of collisions between electrons and atoms, v^*_{en}, is independent of the velocity. If v^*_{en} does depend on the velocity, then in order to obtain congruence between the formulas it is necessary to introduce some averaged collision frequencies into (1.10) and (1.11). The quantity v_{en}, which enters into $D_{\perp}$, is then found to be dependent on the magnetic field. Nevertheless, formula (1.11) correctly expresses the general course of the dependence of the diffusion coefficients on the magnetic field in this case also, if the dependence of v^*_{en} on v is not too pronounced.

The kinetic equation describing the motion of ions in a neutral gas can be easily integrated if the effective cross section of the collisions between an ion and an atom changes in inverse proportion to their relative velocity, $\sigma_{in} \sim 1/v$ (it is assumed that only elastic collisions are important) [18]. For this case, by integrating the kinetic equation, one obtains expressions for the flow which are identical to formulas (1.10)--(1.12), the effective collision frequency being given by the equality.

$$\nu_{in} = \frac{m_n}{m_i + m_n} n_n \int_{(\Omega)} v\sigma_{in}(v, \vartheta)(1 - \cos\vartheta)\, d\Omega. \tag{1.17}$$

3. Dipolar diffusion of charged particles in a weakly ionized gas. The quasi-neutrality of the plasma should be conserved in the course of diffusion of charged particles (if, of course, the dimensions of the inhomogeneous region are much greater than the Debye radius). The quasi-neutrality condition for a plasma made up of electrons and ions with charge Ze has the form

$$n_e = Zn_i. \tag{1.18}$$

Equality (1.18) sets forth the relation between the flows penetrating each element of volume,

$$\nabla(n_e \mathbf{u}_e) = Z\nabla(n_i \mathbf{u}_i). \tag{1.19}$$

In many cases, relation (1.19) determines the absence of an electric current in the direction of the concentration gradient, i.e., the equality of the corresponding components of the directed velocities of electrons and ions

$$u_{e\parallel} = u_{i\parallel}, \ \mathbf{u}_{e\perp} = \mathbf{u}_{i\perp}. \tag{1.20}$$

This equality holds, for example, in the case of diffusion in a dielectric bottle (for more detail, see 3, Ch. I)

Let us note that relations (1.18) and (1.19) do not impose any restrictions on the particle flow in the direction perpendicular to the concentration gradient (it is easy to see that $\nabla(n\mathbf{u}_\perp) = 0$). This is natural, since a flow of this kind does not lead to a change in the particle concentration. The mode of diffusion for which condition (1.20) is valid is termed dipolar.

Let us examine a dipolar diffusion in a weakly ionized gas, when the major part is played by collisions of electrons and atoms with atoms $(\nu_{en} \gg \nu_{ei}, \nu_{in} \gg \nu_{ie})$. The directed velocities of the electrons and ions in this case are given by formulas (1.10)--(1.12). Substituting these formulas into (1.20) and taking (1.18) into account, we can find the electric field strength of the space charge necessary for maintaining the dipolar diffusion*:

$$E_{\parallel} = \frac{(D_{i\parallel} - D_{e\parallel}) \nabla_{\parallel} n_e}{(\mu_{i\parallel} - \mu_{e\parallel}) n_e} \approx - \frac{T_e \nabla_{\parallel} n_e}{e n_e}, \tag{1.21}$$

$$E_{\perp} = \frac{(D_{i\perp} - D_{e\perp}) \nabla_{\perp} n_e}{(\mu_{i\perp} - \mu_{e\perp}) n_e} \approx - \frac{T_e \left[1 - \frac{\omega_e \omega_i T_i}{Z \nu_{en} \nu_{in} T_e} \right] \nabla_{\perp} n_e}{e \left[1 + \frac{\omega_e \omega_i}{\nu_{en} \nu_{in}} \right] n_e}. \tag{1.22}$$

The approximate equalities in (1.21) and (1.22) as well as in the subsequent relations (1.23)--(1.25) are obtained under the conditions $m_i \nu_{in} \gg Z m_e \nu_{en}$, $m_i \nu_{in} \gg T_e / T_i m_e \nu_{en}$, which prevail in virtually all cases, since $m_i \gg m_e$.

It is apparent from formula (1.21) that the longitudinal electric field of the dipolar diffusion accelerates the diffusion of ions and slows down the diffusion of electrons. The transverse electric field, in the case of low magnetic fields, also accelerates the diffusion of ions; with larger fields with which the coefficient of diffusion for ions is greater than for electrons, the sign of the electric field changes.

Using relations (1.10), (1.11), (1.21), (1.22), we find expressions

*The potential electric field given by equalities (1.21) and (1.22) may exist if the distribution of the concentrations is a product of the functions which are dependent on the longitudinal and transverse coordinates $n = n_{\parallel}(r_{\parallel}) n_{\perp}(r_{\perp})$.

for the velocity of the dipolar diffusion of electrons and ions

$$\left.\begin{aligned} u_{en} = u_{in} = -D_{a\parallel}\frac{\nabla_{\parallel} n_e}{n_e}, \\ D_{a\parallel} = \frac{T_i + ZT_e}{m_i\nu_{in} + Zm_e\nu_{en}} \approx \frac{T_i + ZT_e}{m_i\nu_{in}}; \end{aligned}\right\} \quad (1.23)$$

$$\left.\begin{aligned} \mathbf{u}_{e\perp} = \mathbf{u}_{i\perp} = -D_{a\perp}\frac{\nabla_{\perp} n_e}{n_e}, \\ D_{a\perp} = \frac{T_i + ZT_e}{m_i\nu_{in} + Zm_e\nu_{en} + \frac{Zm_e\omega_e^2}{\nu_{en}} + \frac{m_i\omega_i^2}{\nu_{in}}} \approx \frac{T_i + ZT_e}{m_i\nu_{in}\left(1 + \frac{\omega_e\omega_i}{\nu_{en}\nu_{in}}\right)}. \end{aligned}\right\} \quad (1.24)$$

The quantities $D_{a\parallel}$, $D_{a\perp}$ introduced here are called the longitudinal and transverse coefficients of dipolar diffusion.

Using formulas (1.12) and (1.22), one can also readily find the velocity and the direction perpendicular to the concentration gradient in dipolar diffusion:

$$\left.\begin{aligned} \mathbf{u}_{e\top} \approx -\frac{c(T_i + ZT_e)[\mathbf{h}\nabla n_e]}{ZeH\left(1 + \frac{\nu_{en}\nu_{in}}{\omega_e\omega_i}\right)\left(1 + \frac{\nu_{en}^2}{\omega_e^2}\right)n_e}, \\ \mathbf{u}_{i\top} \approx \frac{cm_e\nu_{en}(T_i + ZT_e)[\mathbf{h}\nabla n_i]}{Z^2eHm_i\nu_{in}\left(1 + \frac{\nu_{en}\nu_{in}}{\omega_e\omega_i} + \frac{\omega_i^2}{\nu_{in}^2}\right)n_i}. \end{aligned}\right\} \quad (1.25)$$

These velocities determine the density of the diamagnetic current accompanying the dipolar diffusion

$$\mathbf{j} = Zen_i\mathbf{u}_i - en_e\mathbf{u}_e \approx \frac{c(T_i + ZT_e)}{ZH\left(1 + \frac{\nu_{en}\nu_{in}}{\omega_e\omega_i}\right)}[\mathbf{h}\nabla n_e] \quad (\text{where } \nu_{en} \ll \omega_e). \quad (1.26)$$

4. <u>Diffusion in a completely ionized gas</u>. Let us now examine the diffusion in a completely ionized gas composed of electrons and ions charged Ze.

Let us note that each volume element of a completely ionized gas (when $\nabla H \perp \mathbf{H}$) is acted upon by only two forces: the pressure gradient $\nabla(p_i + p_e)$ and the magnetic pressure gradient $\nabla(H^2/8\pi)$, directed at right angles to the magnetic field. Diffusion, i.e., uniform movement of the gas, can take place provided that these forces are in equilibrium. Therefore, the longitudinal diffusion of a completely ionized gas has no meaning at all, since in the presence of a pressure gradient the longitudinal motion of the plasma is accelerated. The transverse diffusion of a completely ionized gas has meaning if it is considered when the gas as a whole is in equilibrium:

$$-\nabla_{\perp}(p_i + p_e) = \nabla_{\perp}\left(\frac{H^2}{8\pi}\right). \tag{1.27}$$

In the region of diffusion, the magnetic field may be considered homogeneous only if the kinetic pressure is much less than the magnetic pressure:

$$\beta = \frac{8\pi(p_i + p_e)}{H^2} \ll 1. \tag{1.28}$$

The directed transverse motion of electrons and ions is given by the equations (1.7)

$$\left.\begin{aligned} e\mathbf{E} + \frac{e[\mathbf{u}_e\mathbf{H}]}{c} + T_e\frac{\nabla n_e}{n_e} + m_e\nu_{ei}(\mathbf{u}_e - \mathbf{u}_i) = 0, \\ Ze\mathbf{E} + \frac{Ze[\mathbf{u}_i\mathbf{H}]}{c} - T_i\frac{\nabla n_i}{n_i} - m_i\nu_{ie}(\mathbf{u}_i - \mathbf{u}_e) = 0. \end{aligned}\right\} \tag{1.29}$$

When these equations are solved simultaneously, one should keep in mind the quasi-neutrality condition (1.18) and the relationship between the collision frequencies (1.4). Once the system of equations (1.29) has been solved, it is easy to find the directed particle velocities. The velocities of the electrons and ions in the direction of the concentration gradient are identical:

$$\mathbf{u}_{e\perp} = \mathbf{u}_{i\perp} = -D_{\perp}\frac{\Delta_{\perp} n_e}{n_e}, \quad D_{\perp} = \frac{\left(T_e + \frac{1}{Z}T_i\right)\nu_{ei}}{m_e\omega_e^2}, \tag{1.30}$$

i.e., the diffusion of a completely ionized gas across a magnetic field is found to be dipolar, irrespective of the magnitude of the electric field in the plasma. The particle flow in the direction of the electric field is absent.

The velocities of the electrons and ions in the direction perpendicular to the concentration gradient and to the field are given by the equalities

$$\begin{aligned} \mathbf{u}_{e\perp} &= -\frac{c}{H}[\mathbf{h}\mathbf{E}] - \frac{cT_e}{eH}\frac{[\mathbf{h}\nabla n_e]}{n_e}, \\ \mathbf{u}_{i\perp} &= -\frac{c}{H}[\mathbf{h}\mathbf{E}] + \frac{cT_i}{ZeH}\frac{[\mathbf{h}\nabla n_i]}{n_i}. \end{aligned} \tag{1.31}$$

These velocities determine the drift of electrons and ions which exists independently of their collisions with one another.

The density of the diamagnetic current in a completely ionized gas is found to be

$$\mathbf{j} = n_e e (\mathbf{u}_{i\top} - \mathbf{u}_{e\top}) = \frac{c}{ZH}(T_i + ZT_e)[\mathbf{h}\nabla n_e]. \tag{1.32}$$

Formulas (1.30)-(1.32) coincide with the corresponding expression obtained from a kinetic examination with $\omega_e \gg \nu_{ei}$ [20-30], if the effective frequency of the collisions of the electrons and ions is taken to be

$$\nu_{ei} = \frac{4\sqrt{2\pi}}{3} n_i L \left(\frac{Ze^2}{T_e}\right)^2 \left(\frac{T_e}{m_e}\right)^{1/2} \tag{1.33}$$

(L is the so-called "Coulombic logarithm" [6]).

5. Dipolar diffusion in a strongly ionized gas. Let us now consider the diffusion in a gas containing electrons, positive ions of one kind, and neutral particles under conditions where the frequencies of the collisions of the charged particles with the neutral particles and with one another are comparable (a "strongly ionized gas"). The system of equations (1.7) for this case will have the form

$$\left.\begin{aligned} e\mathbf{E} + \frac{e[\mathbf{u}_e\mathbf{H}]}{c} + T_e\frac{\nabla n_e}{n_e} + m_e\nu_{ei}(\mathbf{u}_e - \mathbf{u}_i) + m_e\nu_{en}\mathbf{u}_e = 0, \\ Ze\mathbf{E} + \frac{Ze[\mathbf{u}_i\mathbf{H}]}{c} - T_i\frac{\nabla n_i}{n_i} - m_i\nu_{ie}(\mathbf{u}_i - \mathbf{u}_e) - m_i\nu_{in}\mathbf{u}_i = 0. \end{aligned}\right\} \tag{1.34}$$

Here, the quasi-neutrality of the plasma and the relation between the collision frequency (1.4) have been taken into account. As before, it is assumed that the directed velocity of the atoms is much less than the directed velocity of the charged particles.

A simultaneous solution of equations (1.34) gives the velocities u_e and u_i. We shall not write out the intricate formulas thus obtained. By comparing the velocity components of the electrons and ions in the direction of the concentration gradients $(\mathbf{u}_{\|}, \mathbf{u}_{\perp})$, it is easy to find the dipolar particle velocity in the electric field insuring the dipolar diffusion. The longitudinal and transverse components of these quantities have the form

$$\mathbf{u}_{\parallel} = -D_{a\parallel}\frac{\nabla_{\parallel} n_e}{n_e}, \quad D_{a\parallel} = \frac{T_i + ZT_e}{m_i \nu_{in}}, \tag{1.35}$$

$$\mathbf{E}_{\parallel} = -\frac{T_e}{e}\frac{\nabla_{\parallel} n_e}{n_e}, \tag{1.36}$$

$$\mathbf{u}_{\perp} = -D_{a\perp}\frac{\nabla_{\perp} n_e}{n_e}, \quad D_{a\perp} = \frac{T_i + ZT_e}{m_i \nu_{in}\left[1 + \frac{\omega_e \omega_i}{(\nu_{ei} + \nu_{en})\nu_{in}}\right]}, \tag{1.37}$$

$$\mathbf{E}_{\perp} = -\frac{T_e\left[1 - \frac{T_i}{ZT_e}\frac{\omega_e \omega_i}{(\nu_{ei} + \nu_{en})\nu_{in}}\right]}{e\left[1 + \frac{\omega_e \omega_i}{(\nu_{ei} + \nu_{en})\nu_{in}}\right]}\frac{\nabla_{\perp} n_e}{n_e}. \tag{1.38}$$

We shall also write the formula for the current density in dipolar diffusion (when $\nu_{en} \ll \omega_e$)

$$\mathbf{j} = n_e e(\mathbf{u}_{i\top} - \mathbf{u}_{e\top}) = -\frac{c(T_i + ZT_e)\nabla n_e}{ZH\left[1 + \frac{\nu_{in}(\nu_{ei} + \nu_{en})}{\omega_e \omega_i}\right]}. \tag{1.39}$$

Formulas (1.35)-(1.39) were obtained with the assumption that $m_i\nu_{in} \gg m_e\nu_{en}$. A kinetic treatment of diffusion in a plasma composed of electrons, ions and neutral atoms was given in reference [43] for conditions where the diffusion frequencies of collisions of electrons with atoms and of ions with atoms are independent of the velocity and the inequality $m_e\nu_{ei} \ll m_i\nu_{in}$ applies. The expressions for the velocity of dipolar diffusion and electric field obtained in reference [43] are close to the expression given above if the quantities ν_{en}, ν_{in}, ν_{ei} are those in formulas (1.16), (1.17), and (1.33).

6. <u>Summary of results</u>. Table 1 brings together the formulas for the longitudinal and transverse diffusion coefficients of the electric field insuring dipolar diffusion, and the diamagnetic current of charged particles. The table also includes the condition of applicability of these formulas. These conditions were obtained with the use of inequality (1.6), according to which the directed particle velocity in diffusion

Table 1

	Coefficient of longitudinal diffusion $D_{\parallel}$	Longitudinal electric field with dipolar diffusion $E_{\parallel}$	Coefficient of transverse diffusion $D_{\perp}$	Density of diamagnetic current J	Transverse electric field with dipolar diffusion $E_{\perp}$	Conditions of applicability of formulas
Diffusion of electrons in neutral gas	$\frac{T_e}{m_e \nu_{en}}$	—	$\frac{T_e}{m_e \nu_{en}\left(1+\frac{\omega_e^2}{\nu_{en}^2}\right)}$	$\frac{cT_e\,[\mathbf{h}\nabla n_e]}{H\left(1+\frac{\nu_{en}^2}{\omega_e^2}\right)}$	—	$l_{\parallel} \gg \lambda_e$, $l_{\perp} \gg \lambda_e$ при $\nu_{en} > \omega_e$, $l_{\perp} \gg \bar{\varrho}_e$ при $\omega_e > \nu_{en}$
Diffusion of ions in neutral gas	$\frac{T_i}{m_i \nu_{in}}$	—	$\frac{T_i}{m_i \nu_{in}\left(1+\frac{\omega_i^2}{\nu_{in}^2}\right)}$	$\frac{cT_i\,[\mathbf{h}\nabla n_i]}{H\left(1+\frac{\nu_{in}^2}{\omega_i^2}\right)}$	—	$l_{\parallel} \gg \lambda_i$, $l_{\perp} \gg \lambda_i$ при $\nu_{in} > \omega_i$, $l_{\perp} \gg \bar{\varrho}_i$ при $\omega_i > \nu_{in}$
Dipolar diffusion in weakly ionized gas	$\frac{T_i+ZT_e}{m_i \nu_{in}}$	$-\frac{T_e}{e}\frac{\nabla_{\parallel} n}{n}$	$\frac{T_i+ZT_e}{m_i \nu_{in}\left(1+\frac{\omega_e\omega_i}{\nu_{en}\nu_{in}}\right)}$	$\frac{c(T_i+ZT_e)\,[\mathbf{h}\nabla n_e]}{ZH\left(1+\frac{\nu_{en}\nu_{in}}{\omega_e\omega_i}\right)}$	$-\frac{T_e\nabla_{\perp} n_e\left(1-\frac{T_i\omega_e\omega_i}{ZT_e\nu_{en}\nu_{in}}\right)}{en_e\left(1+\frac{\omega_e\omega_i}{\nu_{en}\nu_{in}}\right)}$	$l_{\parallel} \gg \lambda_i$, $l_{\perp} \gg \lambda_i$ при $\nu_{en}\nu_{in} > \omega_e\omega_i$, $l_{\perp} \gg \bar{\varrho}_e$ при $\omega_e\omega_i > \nu_{en}\nu_{in}$
Diffusion of fully ionized gas	—	—	$\frac{(T_i+ZT_e)\,\nu_{ei}}{Zm_e\omega^2}$	$\frac{c}{ZH}(T_i+ZT_e)\,[\mathbf{h}\nabla n_e]$	—	$l_{\perp} \gg \bar{\varrho}_i \frac{\nu_{ei}}{\omega_e}$ при $\nu_{ei} > \omega_e$, $l_{\perp} \gg \bar{\varrho}_i$ при $\omega_e > \nu_{ei}$
Dipolar diffusion in strongly ionized gas	$\frac{T_i+ZT_e}{m_i \nu_{in}}$	$-\frac{T_e}{e}\frac{\nabla_{\parallel} n}{n}$	$\frac{T_i+ZT_e}{m_i \nu_{in}\left[1+\frac{\omega_e\omega_i}{(\nu_{en}+\nu_{ei})\,\nu_{in}}\right]}$	$\frac{c(T_i+ZT_e)\,[\mathbf{h}\nabla n_e]}{ZH\left[1+\frac{(\nu_{en}+\nu_{ei})\,\nu_{in}}{\omega_e\omega_i}\right]}$	$-\frac{T_e\nabla_{\perp} n_e}{en_e}\times$ $\times\frac{1-\frac{T_i}{ZT_e}\frac{\omega_e\omega_i}{(\nu_{en}+\nu_{ei})\,\nu_{in}}}{1+\frac{\omega_e\omega_i}{(\nu_{en}+\nu_{ei})\,\nu_{in}}}$	Combination of preceding conditions

[при = when]

should be much smaller than the thermal velocity of the particles. When applying inequality (1.6) to a specific case, we utilize the above derived formulas for the components of directed velocity $u_{\parallel}, u_{\perp}, u_{T}$ (the temperatures of the electrons and ions were assumed to be of the same order). In the table, $\frac{|u^{T}\Delta|}{u} \approx T_l$, $\frac{|u^{\parallel}\Delta|}{u} \approx \parallel_l$ are the characteristic dimensions of the homogeneous regions of the plasma; $\lambda_i = \frac{1}{\nu_{in}} \sqrt{\frac{2T_i}{m_i}}$, $\lambda_e = \frac{1}{\nu_{en}} \sqrt{\frac{2T_e}{m_e}}$ are the free-path lengths; $\bar{\varrho}_i = \frac{1}{\omega_i} \sqrt{\frac{2T_e}{m_i}}$, $\bar{\varrho}_e = \frac{1}{\omega_e} \sqrt{\frac{2T_e}{m_e}}$ are the Larmor radaii of the electrons and ions.

2. Transverse Diffusion in a Strong Magnetic Field

1. Initial relations. In this section we shall consider the transverse diffusion of charged particles in strong magnetic fields in which the electrons and ions execute many revolutions about the lines of force of the magnetic field during the period between collisions, and the Larmor radaii of the particles are much smaller than the characteristic dimensions, i.e.,

$$\omega_e \gg \nu_e, \ \omega_i \gg \nu_i, \ \bar{\varrho}_e \ll l_{\perp}, \ \bar{\varrho}_i \ll l_{\perp}. \tag{2.1}$$

Under these conditions the effect of particle collisions on their motion may be considered to be a slight perturbation.

In the absence of collisions, the motion of charged particles in the magnetic field is conveniently represented as a rotation with Larmor frequency about the leading centers. The coordinates of the leading center and the particles are then related as follows

$$\mathrm{R}_\alpha = \mathrm{r}_\alpha + \varrho_\alpha, \ \varrho_\alpha = \frac{cm_\alpha}{Z_\alpha eH} [\mathrm{w}_\alpha \mathrm{h}]. \tag{2.2}$$

Here R_α is the radius-vector of the leading center, r_α is the radius-vector of the particle, P_α is the Larmor radius, w_α is the velocity of rotation.

The leading centers themselves can move with an arbitrary velocity along the magnetic field ($w_{\alpha\|}$). In addition, in a transverse electric field the leading centers drift with the velocity

$$u_E = \frac{c}{H}[\mathbf{E}\mathbf{h}]. \tag{2.3}$$

Thus the vector of the particle velocity is given by the sum

$$v_\alpha = w_{\alpha\perp} + w_{\alpha\|} + u_E. \tag{2.4}$$

The magnitudes of the velocity of rotation of the particles, of the longitudinal velocity, of the rotational phase φ, and of the transverse coordinates of the leading center $R_\perp$ (in the presence of a transverse electric field, i.e., coordinates along this field) are integrals of motion. The particle distribution function in the absence of collisions may be an arbitrary function of the integrals of motion. We shall consider the velocity distribution function w to be Maxwellian and to have a concentration dependent on the transverse coordinates of the leading center*:

$$\left.\begin{aligned} F_\alpha(R_{\alpha\perp}, w_{\alpha\perp}, w_{\alpha\|}, \varphi)\, dw_{\alpha\perp}\, dw_{\alpha\|}\, d\varphi = n_\alpha(R_{\alpha\perp}) f_\alpha(v_\alpha)\, d\mathbf{w}_\alpha, \\ f_\alpha(v_\alpha) = \left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{3/2} e^{-\frac{m_\alpha(v_\alpha - u_E)^2}{2T_\alpha}}. \end{aligned}\right\} \tag{2.5}$$

The distribution (2.5) leads to a known expression for the average drift velocity of the particles acted upon by the concentration gradient.

$$u_{\alpha g} = \int \mathbf{w} n_\alpha(R_{\alpha\perp}) f_\alpha(\mathbf{w}_\alpha)\, d\mathbf{w}_\alpha = \frac{cT_\alpha}{Z_\alpha e H n_\alpha}[\mathbf{h}\nabla n_\alpha]. \tag{2.6}$$

The drift of the leading centers, and correspondingly of the particles themselves, across the magnetic field in the direction of the concentration gradient or of the electric field occurs as a result of the collisions. The particle flow under the assumptions made above may be determined by means of the theory of random wanderings developed by Chandrasekar [46, 12].

Let us direct axis OZ along the magnetic field and axis OX along the concentration gradient. The flow of the leading centers across

*It is easy to show that $dw_\perp dw_\| d\varphi = \frac{1}{w} dw_x dw_y dw_z = \frac{1}{w} d\mathbf{w}.$

plane X = const is given by the obvious equality

$$\Gamma_\alpha = \frac{1}{\Delta t}\left\{\int_0^\infty d(\Delta X)\int_0^{\Delta X} dX'\,[n_\alpha(X-X')W_\alpha(X-X',\,\Delta X) - n_\alpha(X+X')W_\alpha(X+X',\,-\Delta X)]\right\}, \tag{2.7}$$

in which $W_\alpha(X,\Delta X)$ is the probability of displacement during time Δt of the leading center located at point X over distance ΔX. The value of Δt is chosen so that many collisions can take place during this period of time, but so that the average displacement be considerably smaller than the characteristic dimensions $\left(\frac{1}{\nu_\alpha} \ll \Delta t \ll \frac{l^2}{\varrho_\alpha^2 \nu_\alpha}\right)$. We can then represent $n_\alpha W_\alpha$ by the expansion

$$n_\alpha(X-X')W_\alpha(X-X',\,\Delta X) = n_\alpha(X)W_\alpha(X,\,\Delta X) - X'\frac{\partial}{\partial X}(n_\alpha W_\alpha). \tag{2.8}$$

Substituting (2.8) into (2.7), integrating with respect to X', and averaging over the velocities of particles of the "α" kind, we find the following expression for the flow:

$$\Gamma_\alpha = n_\alpha\langle\Delta X_\alpha\rangle - \frac{1}{2}\frac{\partial}{\partial X}[n_\alpha\langle(\Delta X_\alpha)^2\rangle]. \tag{2.9}$$

Here the symbol $\langle\ldots\rangle$ signifies the summation over collisions taking place per unit time, and the averaging over velocities:

$$\langle\Delta X_\alpha\rangle = \frac{1}{\Delta t}\int_{-\infty}^{\infty}\overline{(\Delta X)W_\alpha(X_\alpha,\Delta X)}\,d(\Delta X). \tag{2.10}$$

The averaging is carried out for a fixed value of X_α.

In the case in which the gas contains many types of particles, the transverse flow of particles of each type in the direction of the concentration gradient may of course be represented as a sum of the flows associated with the collisions of particles of a given type with particles of all the other types;

$$\Gamma_\alpha = \sum_{(\beta)}\Gamma_{\alpha\beta},\quad \Gamma_{\alpha\beta} = n_\alpha\langle\Delta X_{\alpha\beta}\rangle - \frac{1}{2}\frac{\partial}{\partial X}[n_\alpha\langle(\Delta X_{\alpha\beta})^2\rangle] \tag{2.11}$$

($\Delta X_{\alpha\beta}$ is the displacement of particles of the "α" kind as a result of collisions with particles of the "β" kind). Therefore we shall subsequently give separate consideration to the diffusion taking place across the magnetic field and associated with various types of collisions.

Under conditions where the Larmor radius of the particles is much smaller than the radius of interaction, the magnetic field has almost no effect on the event of collision itself. Collision under these conditions leads to a change in the velocity of the charged particle by a quantity Δv_α and, according to (2.2) and (2.4), to a displacement of the leading center given by the equality

$$\Delta \mathrm{R}_\alpha = \frac{m_\alpha c}{Z_\alpha eH}[\Delta \mathrm{v}_\alpha \mathrm{h}], \qquad \Delta R_\alpha = \frac{|\Delta v_{\alpha\perp}|}{\omega_\alpha}, \quad \Delta X_\alpha = \frac{m_\alpha c \Delta v_{\alpha y}}{Z_\alpha eH}. \tag{2.12}$$

In collisions in which the relative change of the transverse velocity is appreciable, the leading center is displaced over a length of the order of the Larmor radius (fig. 1).

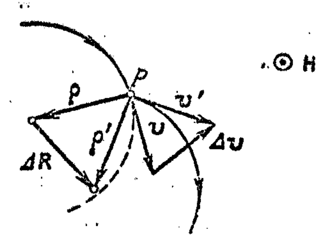

Fig. 1

Summation of the displacements due to collisions of particles of the "α" kind with particles of the "β" kind and averaging over the velocities gives

$$\langle \Delta X_{\alpha\beta}\rangle = \frac{cm_\alpha}{Z_\alpha eH}\int\limits_{(\mathrm{v}_\alpha)} f_\alpha(\mathrm{v}_\alpha)\, d\mathrm{v}_\alpha \int\limits_{(\mathrm{v}_\beta)} n_\beta(X_\beta) f_\beta(\mathrm{v}_\beta)\, d\mathrm{v}_\beta \int\limits_{(\Omega)} (\Delta v_{\alpha y}) v \sigma_{\alpha\beta}(v, \vartheta)\, d\Omega, \tag{2.13}$$

where $v = |v_\alpha - v_\beta|$ is the relative velocity.

The collision integral is calculated in the usual manner [19]

$$\int\limits_{(\Omega)} (\Delta v_{\alpha y}) v \sigma_{\alpha\beta}(v, \vartheta)\, d\Omega = -\frac{m_\beta}{m_\alpha + m_\beta} v_y v s^*_{\alpha\beta}, \tag{2.14}$$

$$s^*_{\alpha\beta} = \int\limits_{(\Omega)} \sigma_{\alpha\beta}(v, \vartheta)(1 - \cos\vartheta)\, d\Omega. \tag{2.15}$$

In the integrals of (2.13) it is convenient to change from the velocities v_α, v_β to the velocities

$$\mathbf{v} = \mathbf{v}_\alpha - \mathbf{v}_\beta, \quad \mathbf{v}_0 = \frac{m_\alpha T_\beta \mathbf{v}_\alpha + m_\beta T_\alpha \mathbf{v}_\beta}{m_\alpha T_\beta + m_\beta T_\alpha}. \tag{2.16}$$

Substituting (2.14) into (2.13) and taking (2.16) into account, we find

$$\langle \Delta X_{\alpha\beta} \rangle = -\frac{\mu_{\alpha\beta} c}{Z_\alpha e H} \int\limits_{(\mathbf{v}_0)} d\mathbf{v}_0 \int\limits_{(\mathbf{v})} d\mathbf{v} f_\alpha(\mathbf{v}_\alpha) f_\beta(\mathbf{v}_\beta) n_\beta(X_\beta) v v_y s^*_{\alpha\beta}(v). \tag{2.17}$$

Here $\mu_{\alpha\beta}$ is the reduced mass

$$\mu_{\alpha\beta} = \frac{m_\alpha m_\beta}{m_\alpha + m_\beta}. \tag{2.18}$$

In formula (2.17) we shall substitute expressions for the distribution functions (2.5), obtained without considering the collisions, since under these conditions the collisions have little effect on the motion of the charged particles (see 2.1).

The expression for $\langle (\Delta X_{\alpha\beta})^2 \rangle$ is obtained in similar fashion;

$$\langle (\Delta X_{\alpha\beta})^2 \rangle = \frac{2}{3} \frac{\mu^2_{\alpha\beta} c^2}{Z^2_\alpha e^2 H^2} n_\beta \int\limits_{(\mathbf{v}_0)} d\mathbf{v}_0 \int\limits_{(\mathbf{v})} d\mathbf{v} f_\alpha(\mathbf{v}_\alpha) f_\beta(\mathbf{v}_\beta) v^3 s_{\alpha\beta}(v). \tag{2.19}$$

In this expression we have neglected the change n_β under the integral sign, i.e., when the coordinates changed by a magnitude of the order of the Larmor radius. Substituting the Maxwell distribution functions in integral (2.19), it is easy to reduce the latter to the form*

$$\langle (\Delta X_{\alpha\beta})^2 \rangle = \frac{8\pi}{3} \frac{\mu^2_{\alpha\beta} c^2}{Z^2_\alpha e^2 H^2} n_\beta \left(\frac{\mu_{\alpha\beta}}{2\pi T_{\alpha\beta}} \right)^{3/2} \int\limits_0^\infty v^5 s^*_{\alpha\beta}(v)\, e^{-\frac{\mu_{\alpha\beta} v^2}{2T_{\alpha\beta}}} dv, \tag{2.20}$$

$$T_{\alpha\beta} = \frac{m_\alpha T_\beta + m_\beta T_\alpha}{m_\alpha + m_\beta}. \tag{2.21}$$

Using formulas (2.11), (2.17) and (2.20) we can determine the flows associated with collisions of particles of various types.

2. Diffusion caused by collisions of charged particles with neutral particles. As before we shall consider that the density of the neutral particles is independent of the coordinates and that the velocity

* When changing to (2.20) we have assumed in formulas (2.5) that v = w, considering that the drift velocity u_e is much less than the thermal velocities $\sqrt{\frac{T_\alpha}{m_\alpha}}, \sqrt{\frac{T_\beta}{m_\beta}}$.

distribution is Maxwellian.

Under these conditions, expression (2.17) for the displacement of charged particles of the "α" kind determined by their collisions with neutral particles, is reduced to the form

$$\Delta X_{an} = \frac{4\pi\mu_{an}^2 c^2 n_n E_x}{3Z_\alpha e H^2 T_{an}} \left(\frac{\mu_{an}}{2\pi T_{an}}\right)^{3/2} \int_0^\infty v^5 s_\alpha^* (\ ) e^{-\frac{\mu_{an} v^2}{2T_{an}}} dv. \tag{2.22}$$

after the substitution of the formula (2.5).

In this case we have left only those terms which are proportional to the first power of E (the drift velocity u_e is assumed to be small compared to the thermal velocity $\sqrt{T_a/m_a}$).

Formulas (2.11), (2.20), and (2.22) make it possible to obtain an expression for the transverse flow charged particles caused by their collisions with neutral particles;

$$\Gamma_{an} = \frac{Z_\alpha \nu_{an}}{m_\alpha \omega_\alpha^2} n_\alpha e E_x - \frac{T_{an} \nu_{an}}{m_\alpha \omega_\alpha^2} \frac{\partial n_\alpha}{\partial X}, \tag{2.23}$$

in which the effective collision frequency $v_{\alpha n}$ is given by the equality

$$\nu_{an} = \frac{8\mu_{an}}{3\sqrt{\pi} m_a} \left(\frac{\mu_{an}}{2T_{an}}\right)^{5/2} \int_0^\infty v^5 s_{\alpha\beta}(v) e^{-\frac{\mu_{\alpha\beta} v^2}{2T_{\alpha\beta}}} dv. \tag{2.24}$$

It should be noted that for the case of electrons, formulas (2.23), (2.24) can be simplified since $m_e \ll m_n$, and therefore (see (2.21))

$$T_{en} \approx T_e, \quad \mu_{en} \approx m_e.$$

The temperature of the ions should be taken to be equal to the temperature of the neutral gas;

$$T_i \approx T_n, \quad T_{in} \approx T_i.$$

Indeed, a complete exchange of the energies of the ions and atoms of comparable masses occurs as a result of several collisions. During that time the ion may be displaced by several Larmor radii, i.e., over a distance considerably smaller than the characteristic dimension (see 2.1). Therefore during the time of the diffusion a thermal equilibrium between the ions and atoms will undoubtedly be established.

Expression (2.23) gives the corresponding values of the diffusion coefficients and of the directed velocity of the particles;

$$\left.\begin{aligned} D_{an} &= \frac{T_\alpha \nu_{an}}{m_\alpha \omega_\alpha^2} = \nu_{an} \bar{\varrho}_\alpha^2, \\ \bar{v}_E &= \frac{Z_\alpha \nu_{an}}{m_\alpha \omega_\alpha^2} eE = -\nu_{an} \left(\frac{m_\alpha c}{Z_\alpha eH}\right) u_E. \end{aligned}\right\} \qquad (2.25)$$

The physical meaning of these expressions is easy to understand. The diffusion coefficient is equal to half the mean square of the particle displacement per unit time. Since the particle displacement is of the order of the Larmor radius ϱ_α in each collision (see Fig. 1), the diffusion coefficient should be of the order of $\nu_{an}\bar{\varrho}_\alpha^2$ (2.25).

A displacement of the particle in the direction of the electric field, in accordance with (2.12) (see also Fig. 1), is given by the velocity change in the perpendicular direction $\Delta X_\alpha = \frac{m_\alpha c}{Z_\alpha eH} \Delta v_{\alpha y}$. Accordingly, the mean velocity in the direction of the field should be proportional to the number of collisions and the mean velocity in the direction perpendicular to the field, $\langle \Delta X_\alpha \rangle = -\nu_{an}\left(\frac{m_\alpha c}{Z_\alpha eH}\right)\bar{v}_{\alpha y}$, as given by equality (2.25).

Using expression (2.23) we can also find, as was done in the preceeding chapter, the velocity of the dipolar diffusion caused by collisions of charged particles with neutral particles. In the case of a gas composed of electrons, one kind of positive ions and neutral particles, the relation giving the dipolar diffusion velocity is obtained by equating Γ_{en} and $\frac{1}{Z}\Gamma_{in}$

$$u_x = \frac{\Gamma_{an}}{n_a} = -D_{a\perp}\frac{\partial n}{\partial X}, \quad D_{a\perp} = \frac{\left(T_e + \frac{1}{Z}T_i\right)\nu_{en}}{m_e\omega_e^2}. \qquad (2.26)$$

The electric field is then given by the approximate formula

$$E_x \approx \frac{T_i}{eZ_i n_e}\frac{\partial n_e}{\partial X}. \qquad (2.27)$$

3. Diffusion caused by collisions of charged particles with one another [35]. Under conditions where the effect of collisions of charged particles with one another on the diffusion is important, the time of the energy exchange of charged particles of various types is found to be considerably shorter than the time of the diffusion. For the energy exchange between electrons and ions, for example, m_i/m_e collisions are required*. The same number of collisions is necessary to make the electrons and ions diffuse over a length of the order of the Larmor ionic radius $\tilde{\rho}_i$ as a result of collisions with one another. Since, according to (2.1), the characteristic dimensions of the plasma are considerably greater, a thermal equilibrium should be established between the charged particles during the period of diffusion. Accordingly, we shall subsequently consider the temperature of charged particles of different types to be the same:

$$T_\alpha = T_\beta = T_{\alpha\beta} = T. \qquad (2.28)$$

Using formula (2.17), let us determine the average displacement in collisions of charged particles, $\Delta X_{\alpha\beta}$. In carrying out the calculation we must consider the dependence of the concentration of particles of the "β" kind on the coordinates:

$$n_\beta(X_\beta) = n_\beta(X_\alpha) + (X_\beta - X_\alpha)\frac{\partial n_\beta}{\partial X}. \qquad (2.29)$$

The difference in coordinates $X_\beta - X_\alpha$ associated with the collisions (for a fixed value of X_α) is determined in accordance with (2.2), (2.4)

* We have in mind the "close" collisions resulting in a change of the electron velocity by an angle $\approx\pi/2$, or a set of distant interaction which is equivalent to a "close" collision.

and (2.16) by the equality

$$X_\beta - X_\alpha = \frac{c}{eH}\left(\frac{m_\beta w_{y\beta}}{Z_\beta} - \frac{m_\alpha w_{y\alpha}}{Z_\alpha}\right) = \\ = \frac{c}{eH}(v_{0y} - u_{Ey})\left(\frac{m_\beta}{Z_\beta} - \frac{m_\alpha}{Z_\alpha}\right) - \frac{c\mu_{\alpha\beta}}{eH} v_y \left(\frac{1}{Z_\alpha} + \frac{1}{Z_\beta}\right). \tag{2.30}$$

Substituting (2.29) and (2.30) into (2.17) we can easily find the magnitude of $\langle\Delta X_{\alpha\beta}\rangle$. Neglecting in (2.29) the terms proportional to the second and higher derivatives of the concentration, since the concentration within the limits of the Larmor radius is small, we obtain the following expression for $\langle\Delta X_{\alpha\beta}\rangle$:

$$\langle\Delta X_{\alpha\beta}\rangle = \frac{4\pi\mu_{\alpha\beta}^2 c^2}{3Z_\alpha e^2 H^2}\left(\frac{1}{Z_\alpha} + \frac{1}{Z_\beta}\right)\frac{\partial n_\beta}{\partial X}\left(\frac{\mu_{\alpha\beta}}{2\pi T}\right)^{3/2}\int_0^\infty v^5 s_{\alpha\beta}^*(v) e^{-\frac{\mu_{\alpha\beta}v^2}{2T}} dv. \tag{2.31}$$

Formulas (2.11), (2.20), and (2.31) give an expression for the diffusion flow caused by the collision of charged particles:

$$\Gamma_{\alpha\beta} = -\frac{\nu_{\alpha\beta} T}{m_\alpha \omega_\alpha^2}\left(\frac{\partial n_\alpha}{\partial X} - \frac{Z_\alpha}{Z_\beta}\frac{n_\alpha}{n_\beta}\frac{\partial n_\beta}{\partial X}\right), \tag{2.32}$$

in which $\nu_{\alpha\beta}$ is given by equality (2.24). Substituting the Rutherford cross section into (2.24) and (2.15), we obtain a relation which gives $\nu_{\alpha\beta}$ in an explicit form

$$\nu_{\alpha\beta} = \frac{4Z_\alpha^2 Z_\beta^2 e^4 (2\pi\mu_{\alpha\beta})^{1/2}}{3m_\alpha T^{3/2}} L n_\beta. \tag{2.33}$$

Here L is the Coulombic logarithm

$$L = \ln\frac{p_{\max}}{p_{\min}}, \qquad p_{\max} = r_d = \left(\frac{T}{4\pi n e^2}\right)^{1/2}, \qquad p_{\min} = r_0 = \begin{cases} r_h = \dfrac{h}{(\mu_{\alpha\beta}T)^{1/2}} & \text{where } r_h \gg r_c, \\ r_c = \dfrac{e^2}{T}\,|Z_\alpha Z_\beta| & \text{where } r_c \gg r_h. \end{cases} \tag{2.34}$$

The expression for the diffusion flow (2.23), taking (2.33) into account, can be represented as

$$\Gamma_{\alpha\beta} = -\frac{4}{3}\left(\frac{2\pi\mu_{\alpha\beta}}{T}\right)^{1/2}\frac{c^2e^2}{H^2} L n_\alpha n_\beta Z_\beta\left(Z_\beta\frac{1}{n_\alpha}\frac{\partial n_\alpha}{\partial X} - Z_\alpha\frac{1}{n_\beta}\frac{\partial n_\beta}{\partial X}\right). \tag{2.35}$$

It should be noted that the collisions of charged particles, independently of the magnitude of the electric field, lead to a dipolar flow in the direction of the concentration gradient. Indeed, according to (2.35)

$$Z_\alpha \Gamma_{\alpha\beta} = -Z_\beta \Gamma_{\beta\alpha}. \tag{2.36}$$

4. On the influence of collisions of like charged particles on the diffusion of the plasma across a magnetic field [35, 39]. It follows from formula (2.35) that collisions between like particles do not result in a diffusion flow proportional to the concentration gradient

$$\Gamma_{\alpha\alpha} = 0. \tag{2.37}$$

In certain studies (for example in ref. [6]) this effect is attributed to the fact that the "center of mass of the leading centers" is displaced during collisions of like particles. Such an explanation should not be considered satisfactory since the diffusion may also take place when the position of the center of the mass of the particles remains unchanged*.

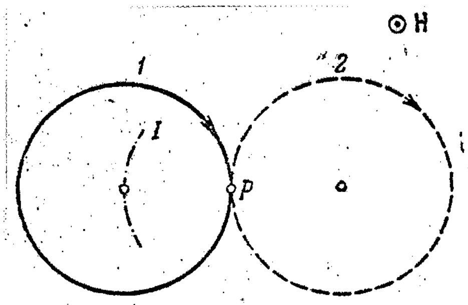

Fig. 2.

As shown in (ref. [35]), in the case of collisions between like particles, the diffusion flow given by the mean square of displacement is balanced by an opposite flow proportional to the mean displacement. The origin of this flow is not difficult to understand. As can be seen from Fig. 2, when particle 1 collides with a similar particle 2, the leading center of particle 1 is displaced toward particle 2 (along curve I). In the presence of a concentration gradient, there are more collisions on the side where the concentration is greater. Therefore, collisions

* Let us note that in accordance with (2.35) the diffusion flow caused by the collision of particles of different masses, in which the center of mass of the leading center is displaced, may also become 0 (for instance if $Z_\alpha = -Z_\beta$, $n_\alpha = n_\beta$).

between like particles lead to an average displacement and correspondingly to their flow in the direction of increasing concentration. This flow offsets the ordinary diffusion flow in the direction of decreasing concentration. A complete equalization of the flows is obtained in accordance with (2.35) when the calculation is made to within the first space derivative of the concentration.

Calculations with higher derivatives of the concentration made it possible to determine the velocity of the diffusion caused by collision of like particles [35]. The corresponding expression for the diffusion flow is

$$\Gamma_{\alpha\alpha}=\frac{8}{15}\pi^{1/2}\frac{c^4m_\alpha^2}{H^4}\left(\frac{T}{m_\alpha}\right)^{1/2}Ln^2\frac{\partial}{\partial X}\left(\frac{1}{n_\alpha}\frac{\partial^2 n_\alpha}{\partial X^2}\right)\div \div\frac{c^4m_\alpha^4}{H^4}\left(\frac{T}{m_\alpha}\right)^{1/2}L\frac{n}{l_\perp^3}. \qquad (2.38)$$

The ratio of the flow of ions caused by their collisions with one another to the flow caused by their collisions with electrons has, in accordance with (2.35) and (2.38), an order of magnitude of

$$\frac{\Gamma_{ii}}{\Gamma_{ie}}\div\left(\frac{m_i}{m_e}\right)^{1/2}\frac{\overline{\rho_i^2}}{l_\perp^2}. \qquad (2.39)$$

(when $n_i \approx n_e$).

In ref. [35] it has been noted that under conditions determined from formula (2.39), the flow of ions due to their collisions with one another may be prevalent. However, as can be seen from (2.38), the collisions of like particles produce different rates of diffusion of electrons and ions (the ions diffuse faster). Therefore, a separation of charges arises in the plasma, and hence the transverse electric field is inhomogeneous in the general case [39]. In an inhomogeneous electric field, the drift velocities of the colliding particles are different (since their leading centers are dispersed in space). The collisions give rise to a "friction force" proportional to the mean relative drift velocity and directed parallel to it (i.e., perpendicular to the electric and magnetic field). The "friction force" gives rise to a particle drift parallel to the electric field. The direction of this drift is opposite for particles of opposite charge. The inhomogeneous electric field thus changes the particle flow due to collisions and may create a dipolar mode of diffusion, i.e., the absence of an electric current in the direction of the gradients. Ref. [39] showed that the electric field produced in dipolar diffusion in a gas composed of electrons and singly charged ions has an order of magnitude of

$$E \div \frac{T}{el_{\perp}} \tag{2.40}$$

(the field distribution is determined by the condition that the transverse current be equal to 0). The ratio of the dipolar diffusion flow proportional to higher derivatives of the concentration Γ^{II} to the flow proportional to the concentration gradient Γ^{I} is given in order of magnitude by the equality

$$\left|\frac{\Gamma^{II}}{\Gamma^{I}}\right| \div \frac{\varrho_i^2}{l_{\perp}^2}. \tag{2.41}$$

Thus the dipolar flow proportional to higher derivatives of the concentration, and in particular, the flow caused by collisions of like particles, are small under the conditions considered (see 2.1) and can be neglected.

5. On the influence of collisions of ions with different charges on the diffusion. If the plasma contains ions with different charges, their collisions may have a considerable effect on the transverse diffusion of the plasma [2, 37]. Indeed, each such collision leads to an appreciable change in the momentum of the ions, i.e., to a displacement of the leading center of the ion over a distance of the order of the Larmor radius while collisions of the ion with an electron result in a

considerably smaller displacement of the ion, since the relative momentum change of the ion is then of the order of the ratio of the electron mass to the ion mass.

The role of the collisions of ions of different charges may be evaluated by means of relation (2.35). For the same relative concentration gradients of the particles $\left(\frac{1}{n_\alpha}\frac{\partial n_\alpha}{\partial X}=\frac{1}{n_\beta}\frac{\partial n_\beta}{\partial X}\right)$, the ratio of the diffusion flow of ions with charge $Z_\alpha e$ caused by collisions with ions possessing charge $Z_\beta e$, to the flow caused by collisions with electrons, becomes the equality

$$\frac{\Gamma_{\alpha\beta}}{\Gamma_{\alpha e}}=\left[\frac{m_\alpha m_\beta}{(m_\alpha+m_\beta)\,m_e}\right]^{1/2}\frac{Z_\beta\,(Z_\beta-Z_\alpha)\,n_\beta}{(Z_\alpha+1)n_e}. \tag{2.42}$$

This ratio may be greater even if $n_\beta \ll n_e$, i.e., if there is present a small amount of impurity ions with a charge different from the charge of the bulk of the ions, since the ion masses m_α, m_β are much greater than the electron masses m_e.

Let us note that in a plasma containing two types of positive ions their collisions lead to an opposite direction of the diffusion flow of ions of different charges (see (2.36)). For the same relative concentration gradient, in accordance with formula (2.35), the ions with the smaller charge should diffuse in the direction in which the concentration decreases (the usual direction of the diffusion), whereas the ions with greater charges should diffuse in the direction in which the concentration increases.

Under steady-state conditions, when the ions are formed in the center of the plasma volume and their concentration decreases toward the periphery, the diffusion of ions of all types should also be directed toward the periphery of the volume. This means that the steady-state distribution of the concentration cannot be characterized by the same relative gradients of ions of both types. It is easy to see that the ions with the greater charge should be more concentrated at the center of the volume than the ions with the smaller charge.

Let us take as an example the diffusion of multicharged ions of an impurity of the "α" kind in a plasma composed mainly of singly charged ions and electrons (assuming that $n_\alpha \ll n_i \approx n_e = n$). The diffusion flow of the impurity ions is given by the equality (2.35)

$$\Gamma_\alpha = \Gamma_{\alpha i} + \Gamma_{\alpha e} = -\frac{4c^2e^2}{3H^2}\left(\frac{2\pi\mu_{\alpha i}}{T}\right)^{1/2} n_\alpha n \left\{\left[\frac{\partial \ln n_\alpha}{\partial X} - Z_\alpha \frac{\partial \ln n}{\partial X}\right] + \right.$$
$$\left. + \left(\frac{m_e}{\mu_{\alpha i}}\right)^{1/2}\left[\frac{\partial \ln n_\alpha}{\partial X} + Z_\alpha \frac{\partial \ln n}{\partial X}\right]\right\}. \quad (2.43)$$

The second term in braces is much smaller than the first since $m_e \ll \mu_{\alpha i}$. Therefore, in order that the flow of the impurity ions be directed toward a decreasing concentration, the inequality

$$\frac{\partial \ln n_\alpha}{\partial X} > Z_\alpha \frac{\partial \ln n}{\partial X}, \quad (2.44)$$

should be obeyed, meaning that the change in the impurity concentration with the coordinate should be faster than n^{Z_α}. This inequality thus determined the sharp drop in the concentration of multicharged impurity ions at the periphery of the plasma.

6. Diffusion in a strong magnetic field affecting particle collisions. Thus far, the diffusion was discussed for conditions under which the magnetic field does not have a direct effect in the collisions, i.e., when the Larmor radii of the particles are much greater than the radius of interaction.

However, in strong magnetic fields for which the radius of interaction of charged particles (the Debye radius) is comparable to the Larmor radius of the electrons or exceeds it, it is necessary to consider the influence of the field on the particle collisions. A discussion of the diffusion of charged particles taking into consideration the direct influence of the magnetic field on the collisions is given in ref. [31, 38]*.

As before, the diffusion flow caused by the collisions can be determined by means of formula (2.11). However the quantities $\langle \Delta X_{\alpha\beta} \rangle$ and $\langle (\Delta X_{\alpha\beta})^2 \rangle$ in this formula should be calculated by taking into consideration the effect of the magnetic field on the motion of the particles. In performing the calculation, we shall assume that the interaction of the charged particles is described by the Coulombic potential "cut off" at the Debye radius.

In order to characterize the collisions in a strong magnetic field, it is convenient to introduce the impact parameters p.

If we determine the relation between the displacement of a particle upon collision $\Delta X_{\alpha\beta}$ and the impact parameter, the average quantity $\langle \Delta X_{\alpha\beta} \rangle$ can be calculated by means of the obvious equality

$$\langle \Delta X_{\alpha\beta} \rangle = \iint\limits_{(p)} dp_1\, dp_2 \int\limits_{(\mathrm{v}_\alpha)} f(\mathrm{v}_\alpha)\, d\mathrm{v}_\alpha \int\limits_{(\mathrm{v}_\beta)} f(v_\beta)\, d\mathrm{v}_\beta n_\beta (X_\beta)\, v (\Delta X_{\alpha\beta}), \tag{2.45}$$

where p_1, p_2 are the projections of the impact parameter on the mutually perpendicular axes in the plane perpendicular to the vector of the relative velocity v.

The quantity $(\Delta X_{\alpha\beta})^2$ is given by a similar equality. The calculation of $\langle \Delta X_{\alpha\beta} \rangle$, $\langle (\Delta X_{\alpha\beta})^2 \rangle$ and of the corresponding diffusion flow for an arbitrary magnetic field effect on the collisions is difficult. For this reason we are considering below two limiting cases. In the first case the motion of one of the particles in one of the collisions is strongly "magnetized", while the second particle is practically unaffected by the magnetic field.

* Ref. [36] also attempts to discuss diffusion in a magnetic field influencing the collisions. This work, however, contains an error: the authors make the incorrect assumption that the total pressure of the particles is constant, without considering the magnetic pressure (for more details, see ref. [38]).

This takes place in the case of collisions of electrons with ions if $\varrho_e \ll p \ll \varrho_i$. In the second case, the motion of both particles is magnetized (collisions of electrons with one another at $\varrho_e \ll p$, collisions of electrons with ions and of ions with one another at $\varrho_i \ll p$).

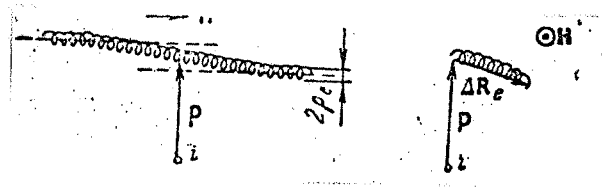

Fig. 3.

7. <u>Diffusion of electrons caused by their collisions with ions at $\varrho_e \ll p \ll \varrho_i$</u>. In a collision process in which the impact parameter is much greater than the Larmor radius, the electron drifts in the electric field of the ion (Fig. 3). The transverse displacement of the leading center of the electron caused by the collision is given by the equality

$$\Delta R_{e\perp} = -\frac{c}{eH}\int_{-\infty}^{\infty} [F_{ei}h]\ dt = \frac{Z_i ec}{H}\int_{-\infty}^{\infty} V[rh]\,dt, \tag{2.46}$$

where $F_{ei} = -Z_i e^2 rV(r)$ is the force exerted on the electron by the ion, and r is the radius vector directed from the ion to the electron

$$V(r) = \frac{1}{r^3} \text{ where } < r_d, \quad V(r) = 0 \text{ where } r > r_d. \tag{2.47}$$

Since the mean thermal velocity of ions is much smaller than that of electrons, we shall first consider the drift of electrons assuming that the ions are at rest. Furthermore, since in the case under consideration the mean energy of the electrons is much greater than the energy of interaction during collisions (since the Larmor radius $\bar{\varrho}_e$ is much greater than the "strong interaction" radius r_c), we shall assume that the longitudinal electron velocity is constant. As follows from (2.46), the projection of the trajectory of the leading center of the electron on plane XY is an arc of a circle with its center at the point where the ion is located (see Fig. 3). The length of the path of the leading center during the time of the collision is given, in accordance with (2.46), by the relation

$$s_e = \frac{cp}{eHv_{ez}} \int_{-\infty}^{\infty} V(r)\, dz \left(r = \sqrt{p^2 + z^2}\right). \tag{2.48}$$

As can be seen from Fig. 3, the displacement vector of the leading center may be determined by the equality

$$\Delta \mathbf{R}_{e\perp} = \frac{s_e [\mathbf{ph}]}{p} - \frac{s_e^2 \mathbf{p}}{2p^2}, \tag{2.49}$$

which allows for the fact that

$$\frac{s_e}{p} \approx \frac{\varrho_e}{p} \frac{r_c}{p} \ll 1; \tag{2.50}$$

the direction of vector p was chosen from the ion to the electron.

Equalities (2.49) and (2.50) give the displacement of the electron in the direction of the gradients

$$\Delta X_e = \frac{cp_y}{eHv_{ez}} \int_{-\infty}^{\infty} V(r)\, dz - \frac{1}{2} \frac{c^2 p_x}{e^2 H^2 v_{ez}^2} \left[\int_{-\infty}^{\infty} (V(r)\, dz) \right]^2. \tag{2.51}$$

When calculating $\langle \Delta X_e \rangle$ by means of (2.45), it is necessary to take into account the change in the concentration of the ions within the region of interaction (this change is assumed to be small)

$$n_i(X_i) = n_i(X_e) - p_x \frac{\partial n_i}{\partial X}. \tag{2.52}$$

The relative velocity which should be substituted into the integrals of (2.45) is obviously the longitudinal velocity of the electron $v = |v_{ez}|$. Calculations using formulas (2.45) and (2.51) and taking (2.52) and (2.5) into consideration produced the following result

$$\langle \Delta X_{eI} \rangle = \left(\frac{2\pi m_e}{T} \right)^{1/2} \frac{c^2 e^2}{H^2} L_p L_v Z_i^2 \frac{\partial n_i}{\partial X}, \tag{2.53}$$

$$L_p = \ln \frac{p_{max}}{p_{min}}, \tag{2.54}$$

$$L_v = 2 \int_{(v)} e^{-\frac{m_e v^2}{2T}} \frac{dv}{v}. \tag{2.55}$$

We can similarly determine the quantity

$$\langle (\Delta X_e)^2 \rangle = 2 \left(\frac{2\pi m_e}{T} \right)^{1/2} \frac{c^2 e^2}{H^2} L_p L_v Z_i^2 n_i. \tag{2.56}$$

The calculation was carried out to within the terms proportional to $1/H^2$ (the terms proportional to higher powers of $1/H$ were omitted). The error which is then permissible is of the order of ρ_e/p.

The velocity integral L_v diverges at the lower (zero) limit. This divergence is due to the fact that in the treatment under consideration, the time of interaction between an electron and an ion increases indefinitely as the longitudinal velocity of the electron decreases. Actually, the interaction time is limited. It cannot be more than $p/v_{i\perp}$ where $v_{i\perp}$ is the transverse velocity component of the ion. When $v_{ez} < v_{i\perp}$, the ion escapes from the region of interaction faster than the electron. Another limitation is associated with the longitudinal acceleration of the electron in the process of collision. The longitudinal electron velocity in the vicinity of the ion cannot be less than the magnitude of the order of

$$v_c = \left(\frac{e^2}{pm}\right)^{1/2} = \bar{v}_e\left(\frac{r_c}{p}\right)^{1/2}. \tag{2.57}$$

Since, however, the integral L_v diverges logarithmically, it is possible to replace the exact calculation by "cutting off" the integral at velocity v_0, equal to v_c or $\bar{v}_i$. We then obtain

$$L_v = \ln\frac{\bar{v}_e^2}{v_0^2} = \begin{cases} \ln\dfrac{m_i}{m_e} \text{ where } \dfrac{m_i}{m_e} < \dfrac{\bar{p}}{r_c}, & (2.58a) \\ \ln\dfrac{\bar{p}}{r_c} \text{ where } \dfrac{\bar{p}}{r_c} < \dfrac{m_i}{m_e}, & (2.58b) \end{cases}$$

$$\bar{p} = (p_{\max}p_{\min})^{1/2}.$$

The expressions for $\langle \Delta X_{eI}\rangle$ and $\langle(\Delta X_e)^2\rangle$ have been derived with the assumption that the ion is at rest. The fact that the ions have a directed velocity caused by the pressure gradient leads to the additional directed electron displacement ΔX_{eII}. The magnitude of this displacement can be determined by means of Einstein's relation between the diffusion coefficients and the mobility of the particles. In order to apply Einstein's relation it is necessary to change to a reference system in which the ions are at rest. This reference system moves relative to the laboratory system at a velocity equal to the velocity of directed motion of the ions u_{ig} (see (2.6)). In the moving system, there arises an electric field whose strength is equal to

$$\mathbf{E}=\frac{1}{c}[\mathbf{u}_{ig}\mathbf{H}]=\frac{T\nabla n_i}{Z_i e n_i}, \quad E_x=\frac{T}{Z_i e n_i}\frac{\partial n_i}{\partial X}, \quad E_y=E_z=0. \tag{2.59}$$

When acted upon by a weak electric field, the electrons should move along the electric field as a result of the collisions, and their average displacement per unit time should be proportional to the field strength:

$$\langle \Delta X_{eII}\rangle=\mu E. \tag{2.60}$$

The electron mobility μ may be determined by means of Einstein's relation

$$\mu=-\frac{e}{T}D. \tag{2.61}$$

The diffusion coefficient entering into this relation is, as we know, determined by the mean square of the displacement

$$D=\frac{1}{2}\langle(\Delta X_e)^2\rangle. \tag{2.62}$$

Using equalities (2.59) - (2.62) we obtain an expression for the mean displacement of electrons which is associated with the directed motion of ions

$$\langle \Delta X_{eII}\rangle=-\frac{1}{2Z_i}\frac{1}{n_i}\frac{\partial n_i}{\partial X}\langle(\Delta X_e)^2\rangle. \tag{2.63}$$

A more detailed treatment of the mechanism of electron displacement associated with the transverse relative motions of the electron and ion, given in ref. [38], has shown that the average displacement $\langle \Delta X_{eII}\rangle$ is chiefly determined by electrons of small longitudinal velocities. The deviations from the Maxwell distribution in the region of low longitudinal velocities may result in a substantial change in $\langle \Delta X_{eII}\rangle$ and hence in the diffusion flow of the particles.

Formulas (2.53), (2.56) and (2.63) determine $\langle\Delta X_e\rangle$ and $\langle(\Delta X_e)^2\rangle$; $\langle\Delta X_e\rangle$ is obtained by summing up $\langle\Delta X_{eI}\rangle$ and $\langle\Delta X_{eII}\rangle$. Using these formulas and (2.11), we obtain the following expressions for the flow of electrons in the direction of the concentration gradient caused by their collisions with ions at $\varrho_e \ll p$:

$$\Gamma_{ei} = -\left(\frac{2\pi m_e}{T}\right)^{1/2} \frac{c^2 e^2}{H^2} L_p L_v n_e n_i Z_i \left(Z_i \frac{1}{n_e} \frac{\partial n_e}{\partial X} + \frac{1}{n_i} \frac{\partial n_i}{\partial X}\right). \tag{2.64}$$

8. <u>Diffusion of ions caused by their collisions with electrons at $\varrho_e \ll p_i \ll \varrho_i$.</u> When $\varrho_i \gg p$, the effect of the magnetic field on the motion of the ion may be neglected. An important result of the collision for the ion is in this case the change in its velocity. The transverse displacement of the leading center of the ion, according to (2.12), is given by the relation

$$\Delta \mathbf{R}_{ie\perp} = \frac{m_i c}{Z_i e H} [\Delta \mathbf{v}_i \mathbf{h}] = \frac{c}{Z_i e H} \int_{-\infty}^{\infty} [\mathbf{F}_{ie} \mathbf{h}]\, dt. \tag{2.65}$$

The integral on the right side of (2.65) gives the impulse acting on the ion in the process of collision. Comparison of relations (2.65) and (2.46) shows that the displacements of the ion and electron are related by the equality

$$\Delta \mathbf{R}_{ie\perp} = \frac{\Delta \mathbf{R}_{ei\perp}}{Z_i}. \tag{2.66}$$

The relation between the displacements $\langle\Delta X_{ie}\rangle$ and $\langle\Delta X_{ei}\rangle$, averaged and summated over collisions, may be determined if equality (2.45) is used:

$$\langle\Delta X_{ie}\rangle = -\frac{1}{Z_i} \langle\Delta X_{ei}\rangle \qquad (n_i \to n_e) \tag{2.67}$$

(the concentration n_i in the expression for $\langle\Delta X_{ei}\rangle$ should be replaced by n_e).

The relation between $\langle(\Delta X_{ie})^2\rangle$ and $\langle(\Delta X_{ei})^2\rangle$ is determined, in accordance with (2.45) and (2.66), by the relation

$$\langle(\Delta X_{ie})^2\rangle = \frac{1}{Z_i^2} \langle(\Delta X_{ei})^2\rangle \quad (n_i \to n_e). \tag{2.68}$$

Using formulas (2.11), (2.67), (2.68), (2.53), (2.62) and (2.56), it is easy to find the diffusion flow of ions in the direction of the concentration gradient caused by their collisions with electrons

$$\Gamma_{ie} = -\left(\frac{2\pi m_e}{T}\right)^{1/2} \frac{c^2 e^2}{H^2} L_p L_v n_e n_i \left(\frac{1}{n_i}\frac{\partial n_i}{\partial X} + Z_i \frac{1}{n_e}\frac{\partial n_e}{\partial X}\right) = \frac{1}{Z_i}\Gamma_{ei}. \tag{2.69}$$

As is apparent from relations (2.64) and (2.69), the collisions between electrons and ions at $\varrho_i \gg p \gg \varrho_e$, as well as $\varrho_e \gg p$, independently of the strength of the electric field, result in a dipolar diffusion flow.

9. Diffusion caused by collisions in which for both of the colliding particle $\varrho \ll p$. In collisions where $\varrho_k \ll p$ for both particles, the latter drift in a direction perpendicular to the magnetic field and joining their lines. The drift paths of the leading centers are illustrated in Fig. 4. When particles with like charges collide, both of the leading centers drift around each other (Fig. 4a). This means that there is an average displacement in the direction of the concentration gradient. Particles of opposite sign and equal charge drift along a straight line in the direction perpendicular to the impact parameter (Fig. 4b). In such collisions, the average displacement parallel to the concentration gradient is equal to zero.

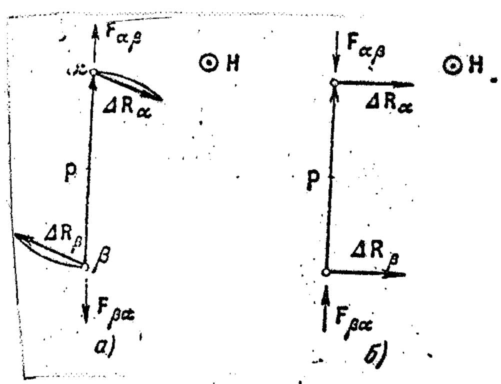

Fig. 4.

The transverse displacement of the leading centers of particles of the "α" kind in the process of their collision with the "β" kind is given by the velocity of their drift:

$$s_\alpha(t) = \frac{c}{Z_\alpha e H}\int_{-\infty}^{t} [\mathbf{F}_{\alpha\beta}\mathbf{h}]\, dt = \frac{Z_\beta e c}{H}\int_{-\infty}^{t} V(r_{\alpha\beta})\,[\mathbf{r}_{\alpha\beta}\mathbf{h}]\, dt. \tag{2.70}$$

Since the interaction of the particles is weak (see p. 31), the components of the vector $r_{\alpha\beta}$ directed from the leading center of particle "β" to the leading center of particle "α" are given by the equalities

$$x_{\alpha\beta} = p_\alpha + s_{\alpha x} - s_{\beta x}, \quad y_{\alpha\beta} = p_y + s_{\alpha y} - s_{\beta y}, \quad z_{\alpha\beta} = v_z t. \qquad (2.71)$$

Since the displacement due to drift is much less than the impact parameter $s \ll p$ (see (2.50)), in calculating the particle displacement with the aid of (2.70) and (2.71), one can make use of the method of successive approximations. To a second approximation (i.e., carrying out the calculations to within the terms proportional to $(s/p)^2$, we obtain the following expression for the displacement of particle "α":

$$= \frac{Z_\beta ec}{Hv_z} p_y \int_{-\infty}^{\infty} V(r_{\alpha\beta})\, dz - \frac{1}{2} \frac{Z_\beta (Z_\alpha + Z_\beta) e^2c^2}{H^2 v_z^2} p_x \left[\int_{-\infty}^{\infty} V(r_{\alpha\beta})\, dz \right]^2. \qquad (2.72)$$

When this expression is used, it is also easy, as was done for other cases, to calculate $\langle \Delta X_{\alpha\beta} \rangle$, $\langle (\Delta X_{\alpha\beta})^2 \rangle$ and the diffusion flow of the particles

$$\Gamma_{\alpha\beta} = -\left(\frac{2\pi\mu_{\alpha\beta}}{T}\right)^{1/2} \frac{c^2c^2}{H^2} L_p L_v n_\alpha n_\beta Z_\beta \left(Z_\beta \frac{1}{n_\alpha} \frac{\partial n_\alpha}{\partial X} - Z_\alpha \frac{1}{n_\beta} \frac{\partial n_\beta}{\partial X} \right). \qquad (2.73)$$

The coefficient L_v in (2.73) is given by formula (2.58b).

From expression (2.73) it is apparent that the diffusion flow caused by collisions of like particles when $\rho \ll p$ as well as in the case when $\rho \gg p$, is absent; the flow due to the average displacement in the direction of the concentration gradient offsets the diffusion flow determined by the mean square of the displacement and directed against the gradient. As in the case where $\rho \gg p$, the change of the flow to zero takes place only when the calculation is carried out to within the first derivative of the concentration. The inclusion of higher derivatives and of the inhomogeneous electric field (see p. 25) gives the flow caused by collisions of like particles.

For a plasma consisting of electrons and ions of one kind, the ratio of the dipolar diffusion flow proportional to higher derivatives of the concentration to the flow proportional to the concentration gradient (the flow caused by collisions of electrons with ions), is given in order of magnitude by the following equalities [39]

$$\frac{\Gamma^{II}}{\Gamma^{I}} \div \frac{\bar{\varrho}_i^2}{l_\perp^2} \text{ where } \varrho_e \ll p \ll \varrho_i, \tag{2.74}$$

$$\frac{\Gamma^{II}}{\Gamma^{I}} \div \frac{p_{max} p_{min}}{l_\perp^2} \text{ where } \varrho_i \ll p. \tag{2.75}$$

Thus, the contribution of collisions of like particles into the diffusion flow is nonessential.

10. Summary of results. Table II gives a summary of the formulas for the transverse flow of charged particles in the direction of the concentration gradient caused by different types of collisions (the formulas are written in the vector form).

The expressions for the flow associated with collisions of charged particles with one another have been given for three cases characterized by different rations of the Debye radius (r_d) to the mean Larmor radii of electrons and ions $\bar{\varrho}_e, \bar{\varrho}_i$.

When $\bar{\varrho}_e \gg r_d$ the diffusion flow is determined by expression (2.35).

In the case where $\bar{\varrho}_i \gg r_d \gg \bar{\varrho}_e$, the diffusion flow caused by the collisions of electrons with ions is the sum of the flow associated with the collisions for which $p < \bar{\varrho}_e$ (this flow is given by formula (2.35)) and of the flow associated with collisions with $p > \bar{\varrho}_e$ (this flow is given by formulas (2.64) and (2.69)).

When $r_d \gg \bar{\varrho}_i$ the diffusion flow of electrons and ions is obtained by summing up the flows for the three ranges of values $p (p < \bar{\varrho}_e, \bar{\varrho}_e < p < \bar{\varrho}_i, p > \bar{\varrho}_i)$, given by formulas (2.35), (2.64), (2.69), and (2.73).

The formulas shown in the table thus enable one to find the diffusion flow of charged particles across a magnetic field in a plasma of various

Table II

Type of Flow	
Flow of charged particles caused by their collisions with neutral particles	$\Gamma_{\alpha n} = -\frac{c^2 m_\alpha T_\alpha \nu_{\alpha n}}{Z_\alpha^2 e^2 H^2} \nabla_\perp n_\alpha + \frac{c^2 m_\alpha \nu_{\alpha n}}{Z_\alpha e H^2} n_\alpha \mathrm{E}_\perp$
Flows caused by collisions of charged particles when $\bar{\varrho} \gg r_d$	$\Gamma_{\alpha\beta} = -\frac{4}{3}\left(\frac{2\pi\mu_{\alpha\beta}}{T}\right)^{1/2} \frac{c^2 e^2}{H^2}\left(\ln\frac{r_d}{r_0}\right) n_\alpha n_\beta Z_\beta \times$ $\times\left(Z_\beta \frac{1}{n_\alpha}\nabla_\perp n_\alpha - Z_\alpha \frac{1}{n_\beta}\nabla_\perp n_\beta\right),$ $\Gamma_{ei} = Z_i \Gamma_{ie} = -\frac{4}{3}\left(\frac{2\pi m_e}{T}\right)^{1/2} \frac{c^2 e^2}{H^2}\left(\ln\frac{r_d}{r_0}\right) n_e n_i Z_i \times$ $\times\left(Z_i \frac{1}{n_e}\nabla_\perp n_e + \frac{1}{n_i}\nabla_\perp n_i\right)$
Flows caused by collisions of electrons with ions when $\bar{\varrho}_e \ll r_d \ll \bar{\varrho}_i$	$\Gamma_{ei} = Z_i \Gamma_{ie} = -\left(\frac{2\pi m_e}{T}\right)^{1/2} \frac{c^2 e^2}{H^2} \times$ $\times\left(\frac{4}{3}\ln\frac{\bar{\varrho}_e}{r_0} + \ln\frac{r_d}{\bar{\varrho}_e}\ln\frac{\bar{v}_e^2}{v_0^2}\right) n_e n_i Z_i \times$ $\times\left(Z_i \frac{1}{n_e}\nabla_\perp n_e + \frac{1}{n_i}\nabla_\perp n_i\right), \quad \frac{\bar{v}_e^2}{v_0^2} = \frac{m_i}{m_e}$ or $\frac{(\bar{\varrho}_e r_d)^{1/2}}{r_c}$ (equal to the smaller of the values)
Flows caused by collisions of charged particles when $\bar{\varrho} \ll r_d$	$\Gamma_{ee} = 0,$ $\Gamma_{ei} = Z_i \Gamma_{ie} = -\left(\frac{2\pi m_e}{T}\right)^{1/2} \frac{c^2 e^2}{H^2}\left(\frac{4}{3}\ln\frac{\bar{\varrho}_e}{r_0} + \frac{1}{2}\ln\frac{m_i}{m_e} \times\right.$ $\left.\times \ln\frac{\bar{v}_e^2}{v_0^2} + \ln\frac{r_d}{\bar{\varrho}_i}\ln\frac{(r_d\bar{\varrho}_i)^{1/2}}{r_c}\right) \times$ $\times n_e n_i Z_i\left(Z_i \frac{1}{n_e}\nabla_\perp n_e + \frac{1}{n_i}\nabla_\perp n_i\right),$ $\Gamma_{\alpha\beta} = -\left(\frac{2\pi\mu_{\alpha\beta}}{T}\right)^{1/2} \frac{c^2 e^2}{H^2}\left(\frac{4}{3}\ln\frac{\bar{\varrho}_i}{r_d} + \ln\frac{r_d}{\bar{\varrho}_i} \times\right.$ $\left.\times \ln\frac{(r_d\bar{\varrho}_i)^{1/2}}{r_c}\right) n_\alpha n_\beta Z_\beta\left(Z_\beta \frac{1}{n_\alpha}\nabla_\perp n_\alpha - Z_\alpha \frac{1}{n_\beta}\nabla_\perp n_\beta\right).$ (the "α" and "β" particles are ions)

compositions. Using these formulas (see also (2.26)) it is easy to obtain an expression for the dipolar flow of charged particles in a plasma composed of electrons, one kind of ions and neutral particles $(n_e = Zn_i)$:

$$\Gamma_{e\perp} = \Gamma_{en} + \Gamma_{ei} = -D_{a\perp}\nabla_\perp n_e, \tag{2.76}$$

$$\Gamma_{i\perp} = \Gamma_{ie} + \Gamma_{i\,i} = -D_{a\perp}\nabla_\perp n_i, \tag{2.77}$$

$$D_{a\perp} = \frac{Z_i+1}{Z_i}\frac{T}{m_e\omega_e^2}(\nu_{en}+\nu_{ei}), \tag{2.78}$$

$$\nu_{ei} = \frac{4}{3}(2\pi)^{1/2}\frac{Z_i^2 e^4 n_i}{(m_e)^{1/2}T^{3/2}}\Lambda, \tag{2.79}$$

$$\Lambda = \begin{cases} \ln\dfrac{r_d}{r_0} & \text{at } r_d \ll \bar{\varrho}_e, \quad (2.80a) \\ \ln\dfrac{\bar{\varrho}_e}{r_0} + \dfrac{3}{4}\ln\dfrac{r_d}{\bar{\varrho}_e}\ln\dfrac{\bar{v}_e^2}{v_0^2} & \text{at } \bar{\varrho}_e \ll r_d \ll \bar{\varrho}_i, \quad (2.80b) \\ \ln\dfrac{\bar{\varrho}_e}{r_0} + \dfrac{3}{8}\ln\dfrac{m_i}{m_e}\ln\dfrac{\bar{v}_e^2}{v_0^2} + \dfrac{3}{4}\ln\dfrac{r_d}{\bar{\varrho}_i}\ln\dfrac{(r_d\bar{\varrho}_i)^{1/2}}{r_c} & \text{at } r_d \gg \bar{\varrho}_i; \quad (2.80c) \end{cases}$$

$\dfrac{\bar{v}_e^2}{v_0^2}$ is equal to the smaller of the values $\dfrac{m_i}{m_e}$ and $\dfrac{\bar{\varrho}_e r_d}{r_c}$.

We have introduced here the coefficient of dipolar diffusion $D_{\alpha\perp}$ determined by the effective frequencies of collisions between electrons and atoms (relation (2.24)) and with ions (2.79). It has been assumed in the calculation that the temperatures of the electrons and ions are the same (see p. 22).

3. Solution of Certain Diffusion Problems

Given below are the solutions of a series of boundary value problems for the diffusion of a plasma in a magnetic field. These solutions are necessary for an analysis of the experimental results.

1. Diffusion equations and boundary conditions. In this chapter we shall consider a plasma composed of electrons, singly charged ions and neutral particles. The electron and ion temperature will be assumed invariant in the process of diffusion (constant in space and time).

The change in the plasma concentration ($n = n = n_e$) is determined by the diffusion flow of charged particles ($\Gamma = nu$) and by the processes of ionization and neutralization in the bulk:

$$\frac{\partial n}{\partial t} + \nabla\Gamma = \frac{\delta n}{\delta t}. \tag{3.1}$$

It is evident that because of the quasi-neutrality of the plasma, the flows of the electrons and ions are related as follows:

$$\nabla\Gamma_e = \nabla\Gamma_i. \tag{3.2}$$

The change in the concentration of charged particles associated with bulk processes, $\delta n/\delta t$, may be represented by the sum

$$\frac{\delta n}{\delta t} = z_i n - \alpha n^2, \tag{3.3}$$

the first term of which gives the rate of ionization and the second, the rate of the electron-ion recombination. The average frequency of ionization z and recombination coefficient are assumed to be independent of the concentration. Formula (3.2) does not allow for the change in the plasma concentration due to the capture of electrons by neutral atoms and subsequent ionic recombination. Therefore, the results obtained by means of equations (3.1) and (3.3) are valid for the analysis of diffusion in gases having a small effective cross section of negative ions (for instance in "electro-positive" inert gases)*.

We shall now apply the diffusion equation (3.1) to three special cases.

a) Diffusion of charged particles of a weakly ionized gas in a bottle with dielectric walls. In the case where the plasma is bounded by dielectric walls the flows of electrons and ions should be identical in all parts of the walls. It is therefore possible to seek a solution of equation (3.1) and (3.2) for which the flows of electrons and ions in the direction of the concentration gradient are the same over the entire

* Let us note that the extension of the results obtained in this section to the case where the process of capture of electrons is essential may be carried out without difficulty. We do not cite the corresponding formulas so as not to encumber the paper.

volume:

$$\Gamma_{e\parallel} = \Gamma_{i\parallel}, \qquad \Gamma_{e\perp} = \Gamma_{i\perp}. \tag{3.4}$$

i.e., the diffusion is dipolar (see p. 7). Substituting expressions (1.23) and (1.24) determining the dipolar diffusion flow, into (3.1), we get the equation of dipolar diffusion

$$\frac{\partial n}{\partial t} - D_{a\parallel} \frac{\partial^2 n}{\partial z^2} - D_{a\perp} \Delta_\perp n = z_i n. \tag{3.5}$$

Here $\Delta_\perp n = \frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2}$; it is assumed that the recombination in the weakly ionized gas is not important.

When a current flows through the plasma, the equation of dipolar diffusion (3.5) is also valid if the density of the electron and ion flows can be represented as a sum of the "flow" components, whose divergence is equal to zero and the "diffusion" components, related by equality (3.4).

The distribution of the electric field in dipolar diffusion is given by relations (1.21) and (1.22)

$$E_z = \frac{D_{i\parallel} - D_{e\parallel}}{\mu_{i\parallel} - \mu_{e\parallel}} \frac{1}{n} \frac{\partial n}{\partial z}, \qquad \mathbf{E}_\perp = \frac{D_{i\perp} - D_{e\perp}}{\mu_{i\perp} - \mu_{e\perp}} \frac{1}{n} \nabla_\perp n. \tag{3.6}$$

The condition for the existence of a potential electric field (rot E = 0) determined by these relations is the equality

$$\frac{\partial^2 \ln n}{\partial x\, \partial z} = \frac{\partial^2 \ln n}{\partial y\, \partial z} = 0, \tag{3.7}$$

from which it follows that the dipolar diffusion in a magnetic field can be present only if the space distribution of the concentration is a product of the functions dependent on the longitudinal and transverse coordinates:

$$n(x, y, z) = n_\parallel(z)\, n_\perp(x, y). \tag{3.8}$$

We shall subsequently consider only such distributions.

In accordance with relations (3.6), the potential of the dielectric walls of the bottle containing a plasma in a magnetic field is found to be non-uniform. In a strong magnetic field, the walls of the bottle perpendicular to the magnetic field are charged negatively (since the diffusion coefficient of electrons along the magnetic field is much greater than the diffusion coefficient of ions) while the walls parallel to the field are charged positively (the coefficient of transverse diffusion of electrons is much greater than that of the ions).

b) <u>Diffusion of charged particles of a weakly ionized gas in a bottle with conducting walls</u>. In the case where the walls of the bottle are made of a high-conductivity material, the potentials of the walls become equal, and consequently, the distribution of the electric field in the volume changes and the dipolar mechanism of diffusion is correspondingly disturbed. This effect was first noted by Simon [47, 48] who called it the "short-circuit effect". The diffusion of a weakly ionized gas in a bottle with conducting walls should be treated by means of equations obtained after substituting into (3.1) and (3.2) the relation for the flows (1.10) and (1.11):

$$\frac{\partial n}{\partial t} - D_{e\parallel}\left[\frac{\partial^2 n}{\partial z^2} + \frac{e}{T_e}\frac{\partial}{\partial z}(nE_z)\right] - D_{e\perp}\left[\Delta_\perp n + \frac{e}{T_e}\nabla_\perp(n\mathbf{E})\right] = z_i n, \qquad (3.9)$$

$$D_{e\parallel}\left[\frac{\partial^2 n}{\partial z^2} + \frac{e}{T_e}\frac{\partial}{\partial z}(nE_z)\right] + D_{e\perp}\left[\Delta_\perp n + \frac{e}{T_e}\nabla_\perp(n\mathbf{E})\right] =$$
$$= D_{i\parallel}\left[\frac{\partial^2 n}{\partial z^2} - \frac{e}{T_i}\frac{\partial}{\partial z}(nE_z)\right] + D_{i\parallel}\left[\Delta_\perp n - \frac{e}{T_i}\nabla_\perp(n\mathbf{E})\right]. \qquad (3.10)$$

These equations must be supplemented with the conditions of equi-potentiality of the walls (i.e., plasma boundaries).

We shall subsequently seek the solutions of equations (3.9) and (3.10) considering that, as in the case of dipolar diffusion, the components of the electric field are proportional to the components of the concentration gradient:

$$E_z = \xi \frac{1}{n}\frac{\partial n}{\partial z}, \qquad \mathbf{E}_\perp = \xi \frac{1}{n}\nabla_\perp n. \qquad (3.11)$$

If we assume that the boundary concentration of the plasma is constant, the equipotentiality condition (in contrast to the case of dipolar diffusion) leads to the identity of the coefficients ξ in the expressions for E_z and $E_\perp$ *.

Substituting (3.11) into (3.9) and (3.10) we obtain

$$\frac{\partial n}{\partial t} - D_{e\parallel}\left(1 + \frac{e}{T_e}\xi\right)\frac{\partial^2 n}{\partial z^2} - D_{e\perp}\left(1 + \frac{e}{T_e}\xi\right)\Delta_\perp n = z_i n, \qquad (3.12)$$

$$\left(1 + \xi\frac{e}{T_e}\right)\left(D_{e\parallel}\frac{\partial^2 n}{\partial z^2} + D_{e\perp}\Delta_\perp n\right) = \left(1 - \xi\frac{e}{T_i}\right)\left(D_{i\parallel}\frac{\partial^2 n}{\partial z^2} + D_{i\perp}\Delta_\perp n\right). \qquad (3.13)$$

These equations will be used in the analysis of diffusion in bottles with metallic walls.

c) Transverse diffusion in a strong magnetic field. In considering transverse diffusion, we assume that the plasma concentration is independent of coordinate z directed along the magnetic field. In this case, the diffusion may be dipolar independently of the walls bounding the plasma. Indeed, according to equality (1.22), in transverse dipolar diffusion, the potential at any point is uniquely determined by the plasma concentration. Therefore the potential of the walls is the same regardless of their conductivity. Substituting into (3.1) the expression (2.78) for the coefficient of dipolar diffusion we obtain an equation describing the transverse diffusion in a strong magnetic field:

$$\frac{\partial n}{\partial t} - D^0_{a\perp}\nabla_\perp[(1+\gamma n)\nabla_\perp n] = z_i n - \alpha n^2. \qquad (3.14)$$

In equation (3.14) $D^0_{a\perp}$ is the coefficient of dipolar diffusion due to collisions of charged particles with neutral ones (the "linear part" of the diffusion coefficient),

$$\gamma n = \frac{\nu_{ei}}{\nu_{en}}, \qquad (3.15)$$

and $D_{ei} = \gamma n D^0_{a\perp}$ determines the coefficient of diffusion due to collisions of electrons with ions. The coefficient γ in accordance with (2.79)

* The variation of the boundary concentration within certain limits is not essential, since the potential of the walls in accordance with (3.11) shows a logarithmic dependence on the boundary concentration.

has a logarithmic dependence on the concentration. This weak dependence of γ on n will henceforth not be considered.

In order to make the solution of the equation obtained definite, it is necessary to establish the boundary values of the plasma concentration. The concentration values in the vicinity of the absorbing walls in the absence of a magnetic field have been determined in many studies (see, for example, [49]). In ref. [50], an analysis is given for certain cases of the influence of the transverse magnetic field on the boundary conditions. We shall not give an account of the results of these studies. It is sufficient to indicate that in most cases where a diffusion treatment was applicable (see Table I), the concentration of the plasma in the vicinity of the absorbing wall was found to be substantially smaller than the concentration in the central region. Therefore, in solving the diffusion equations (3.5), (3.12), (3.13) and (3.14), the concentration at the boundary (in the vicinity of the walls of the bottle) will be assumed to be zero:

$$n_{\text{rp}} = 0; \tag{3.16}$$

2. <u>Steady "diffusion" conditions of a plasma</u>. As we know, the steady diffusions of a weakly ionized gas is achieved in a "long" positive discharge column in which the magnetic field is directed along the axis. Equation (3.5) for this case assumes the form

$$\Delta_{\perp} n + \frac{z_i}{D_{a\perp}} n = 0. \tag{3.17}$$

The distribution of the concentrations is given by the non-negative solution of equation (3.17) which becomes zero at the boundaries. (Such a distribution is called a "diffusion" distribution). The characteristic relation

$$z_i = \frac{D_{a\perp}}{\Lambda_0^2}, \tag{3.18}$$

should then be fulfilled, which represents the equilibrium condition, i.e., the equality of the rates of formation and "removal" of the charged particles. The quantity Λ_o is of the order of the transverse dimensions of the plasma container and is called the "diffusion length." When the plasma bottle is cylindrical in shape, the diffusion distribution and the diffusion length are given by

$$n = n_0 J_0\left(\frac{r}{\Lambda_0}\right), \qquad \Lambda_0 = \frac{a}{2,405}, \tag{3.19}$$

where J_o is a Bessel function and a is the radius of the bottle.

The quantities Z_1 and $D_{\alpha\perp}$ are uniquely related to the electron temperature. Therefore relation (3.18) can be used for its calculation. Since, further, the electron temperature in the positive discharge column is determined by the longitudinal electric field, the magnitude of this field can be calculated as well as the steady plasma concentration, which is inversely proportional to the electric field strength.

As the magnetic field increases as a result of the decrease in the diffusion coefficient, the necessary electron temperature and longitudinal electric field strength should decrease (see Fig. 14). The plasma concentration should correspondingly increase when the discharge current is fixed.

Thus, by determining experimentally the change of the electron temperature with the magnetic field and the change in the electric field strength or plasma concentration in the positive discharge column, one can estimate the influence of the magnetic field on the coefficient of transverse diffusion. It should be noted that T_e, E_z, n_o change relatively slowly with a change in $D_{\alpha\perp}$. A two to three-fold change in $D_{\alpha\perp}$ causes a 10-30% change in E_z, n_o.

3. Diffusion of a stationary charge from the plasma. Under conditions when the region of the active discharge occupies a part of the discharge bottle, there occurs a diffusion of the charged particles from this region. At some distance from the active region of the discharge, there can be established a low electron temperature at which there is no gas ionization. We shall consider below the diffusion of charged particles from a stationary discharge into a region where there is no ionization, using several cases.

a) Weakly ionized gas; dielectric bottle; boundary of the active region parallel to the magnetic field (Fig. 5). The diffusion equation (3.5) for the case under consideration $\left(\frac{\partial}{\partial t}=0,\ z_i=0\right)$ assumes the form

$$D_{a\parallel}\frac{\partial^2 n}{\partial z^2}+D_{a\perp}\Delta_\perp n=0. \tag{3.20}$$

We shall cite the solution for this equation of type (3.8); this solution becomes zero at the boundaries and rises monotonically from the boundaries to the center of the volume. When the boundary of the diffusion region and the walls of the bottle are planar, this solution is given by the equality

$$n(x,z)=A\sin\frac{\pi z}{d}\,\mathrm{sh}\left[\frac{\pi}{d}\sqrt{\frac{D_{a\parallel}}{D_{a\perp}}}\,(a_2-x)\right]. \tag{3.21}$$

If $\sqrt{\frac{D_{a\parallel}}{D_{a\perp}}}\frac{\pi(a_2-x)}{d}\gg 1$, the hyperbolic sine can be replaced by the exponent:

$$n(x,z)\approx B\sin\frac{\pi z}{d}\,e^{-\frac{x}{s_\perp}}, \tag{3.22}$$

$$s_\perp=\frac{d}{\pi}\sqrt{\frac{D_{a\perp}}{D_{a\parallel}}}. \tag{3.23}$$

The solution obtained gives the transverse decrease in concentration if the longitudinal distribution at the boundary of the active discharge region is sinusoidal $\left(\sin\frac{\pi z}{d}\right)$. It can be assumed that a concentration distribution close to (3.22) is established at a sufficiently great distance from the boundary of the active region (where $\frac{x-a_1}{s_\perp} \gg 1$) for other boundary distributions as well.

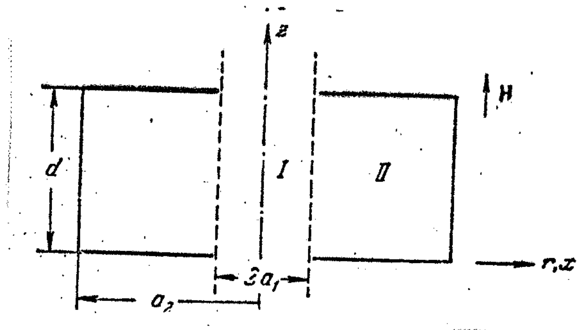

Fig. 5. Diagram of the discharge volume.

I. Active discharge region
II. Diffusion region.

The solution of equation (3.20) for a cylindrical shape of the boundary of the active discharge and a cylindrical shape of the bottle is found in similar fashion. When $(a_2-r) \gg s_\perp$, $(r-a_1) \gg s_\perp$, this solution has the form

$$n(r,z) = B \sin\frac{\pi z}{d} \frac{e^{-\frac{r}{s_\perp}}}{\sqrt{r}}. \qquad (3.24)$$

Thus, at a sufficient distance from the boundary of the "active" discharge region and from the walls of the container, the plasma concentration drops exponentially. The characteristic length of the "concentration" $S_\perp$ (the length at which the concentration decreases e times) is of the order of magnitude of the distance to which the charged particles are able to diffuse across the magnetic field during their lifetime. Indeed, relation (3.23) for $S_\perp$ may be rewritten as follows:

$$s_\perp = \sqrt{D_{a\perp}\tau_\parallel}, \quad \tau_\parallel = \frac{d^2}{\pi^2 D_{a\parallel}}. \qquad (3.25)$$

The lifetime of the charged particles in the case under consideration is equal to the time of their longitudinal diffusion.

b) Weakly ionized gas; metallic bottle; boundary of the plasma parallel to the magnetic field. As already indicated, the dipolar mechanism in a magnetic field is disturbed in a bottle with conducting walls.

The diffusion of the stationary charge from the plasma was studied for this case in the work of Simon [47, 48] by means of equations (3.9) and (3.10). In his treatment, Simon neglected in these equations the terms containing the transverse electric field $E_{\perp}$. As was indicated in the work of Zharinov and Tonks [51, 52] such an omission is not justified. It is readily seen that the terms containing $E_{\perp}$ in equations (3.9) and (3.10) are of the same order as the terms describing the transverse diffusion. In reference [52] a correct analysis was made of the diffusion from the plasma of a stationary discharge in a magnetic field (for the planar case) for various boundary conditions corresponding to dielectric and metallic container walls. However, concrete results were obtained in this study only for a definite relation between the longitudinal and transverse dimensions of the bottle.

It is easy to treat the diffusion in a bottle with metallic walls by means of equations (3.12) and (3.13). For the diffusion of a stationary discharge from the plasma $(\frac{\partial}{\partial t}=0,\ z_i=0)$, these equations may be represented as

$$\left.\begin{aligned}\left(1+\frac{e}{T_e}\xi\right)\left(D_{e\parallel}\frac{\partial^2 n}{\partial z^2}+D_{e\perp}\Delta_{\perp}n\right)=0,\\ \left(1-\frac{e}{T_i}\xi\right)\left(D_{i\parallel}\frac{\partial^2 n}{\partial z^2}+D_{i\perp}\Delta_{\perp}n\right)=0.\end{aligned}\right\} \tag{3.26}$$

Two groups of solution for equations (3.26) are possible, corresponding to the positive and negative potential of the metallic walls. In most cases, the walls should acquire a negative charge since the electrons reach them faster than the ions. Equations (3.26) are then reduced to the following equality

$$D_{i\parallel}\frac{\partial^2 n}{\partial z^2}+D_{i\perp}\Delta_{\perp}n=0, \tag{3.27}$$

$$\xi=-\frac{T_e}{e}. \tag{3.28}$$

According to (3.11) the electric field strength is given by the relation

$$\mathbf{E} = -\frac{T_e \nabla n}{en}. \tag{3.29}$$

It is easy to see (see (1.10) and (1.11)) that the field distribution given by these relations leads to the absence of electron flow in the whole region of diffusion. The electrons are "locked" in this region owing to the negative potential of the walls.* The distribution of the concentration of charged particles is given by equation (3.27) which describes the diffusion of ions. This equation is the same as equation (3.20). Therefore the formulas (3.21)-(3.25) obtained above may be applied to the case under consideration if their coefficients of dipolar diffusion are replaced by the diffusion coefficients of the ions. In particular, the characteristic length of the concentration drop associated with diffusion in a metallic bottle is given by the equality (cf. (3.23) and (3.25))

$$s_\perp = \sqrt{D_{i\perp}\tau_{i\parallel}} = \frac{d}{\pi}\sqrt{\frac{D_{i\perp}}{D_{i\parallel}}}. \tag{3.30}$$

c) <u>Weakly ionized gas; dielectric bottle; boundary of the active discharge region perpendicular to the magnetic field (Fig. 6)</u>. The solution of equation (3.20) for cylindrically symmetrical boundaries in which we are interested is

$$n(r, z) = AJ_0\left(\frac{r}{\Lambda_0}\right) \operatorname{sh}\left[\sqrt{\frac{D_{a\perp}}{D_{a\parallel}}}\frac{(d-z)}{\Lambda_0}\right], \tag{3.31}$$

or when

$$(d-z) \gg \Lambda_0 \sqrt{\frac{D_{a\parallel}}{D_{a\perp}}} \tag{3.32}$$

$$n(r, z) = BJ_0\left(\frac{r}{\Lambda_0}\right) e^{-\frac{z}{s_\parallel}}, \tag{3.33}$$

$$s_\parallel = \sqrt{\frac{D_{a\parallel}}{D_{a\perp}}}\Lambda_0 = \sqrt{D_{a\parallel}\tau_\perp} \quad \left(\tau_\perp = \frac{\Lambda_0^2}{D_{a\perp}}\right).$$

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*Let us note that the total number of electrons and the total number of ions which then reach an isolated metallic wall should be the same under steady conditions. However, the electrons should reach the wall outside of the region under consideration, i.e., in the region of the active discharge or at the boundary of the diffusion region.

The characteristic length $S_{\parallel}$ is determined in this case by the distance to which the charged particles are able to diffuse along the magnetic field during their lifetime (equal to the time of transverse diffusion).

By measuring the value of $S_{\parallel}$, it is of course possible to determine the relation $D_{a\perp}/D_{a\parallel}$. It should be noted that in a strong magnetic field, the length S may be much greater than the transverse dimensions of the bottle.

Reference [53] discusses the solution of the diffusion equation (3.5) for the more complex case where the plasma concentration at the boundary of the active region periodically changes with time.

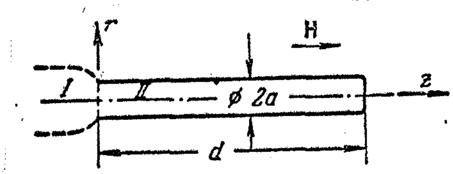


Fig. 6. Diagram of the discharge volume.

I. Active discharge regions;
II. Diffusion region.

In this case as well, the decrease in the concentration averaged over time and the change in the oscillation phase of the concentration along the length of the bottle are determined by the ratio of the diffusion coefficients $D_{a\perp}/D_{a\parallel}$.*

d) Transverse diffusion in a strongly ionized gas at a large magnetic field. Under conditions where the main influence on the diffusion is exerted by the collisions of charged particles with one another,

*After the present survey was written, a paper was published by Golubev and Granovskii [118] which described an experimental study of diffusion in a cylindrical bottle for a constant and a periodically changing concentration at the boundary of the diffusion region. In this study, the distribution of the concentration over the length of the bottle was determined by sounding. Data were obtained on the diffusion of charged plasma particles in helium and argon at magnetic fields up to 1500 Oe The data obtained confirmed the feasibility of using the method proposed by the authors in reference [53] for the study of diffusion.

the transverse diffusion from a stationary plasma is described by equation (3.14) in which it is necessary to put $\frac{\partial n}{\partial t} = 0$, $z_i = 0$, $\gamma n \gg 1$:

$$\frac{1}{2}\gamma D^0_{a\perp}\Delta_\perp(n^2) - \alpha n^2 = 0. \tag{3.34}$$

The solution of this equation for a cylindrical shape of the boundaries (which become zero when r = a_2) may be written as

$$n^2(r) = A\left[I_0\left(\frac{2a_2}{s_\perp}\right)K_0\left(\frac{2r}{s_\perp}\right) - K_0\left(\frac{2a_2}{s_\perp}\right)I_0\left(\frac{2r}{s_\perp}\right)\right], \tag{3.35}$$

$$s_\perp = \sqrt{\frac{\gamma D^0_{e\perp}}{\alpha}} = \sqrt{\frac{D_{ei}}{\alpha n}}. \tag{3.36}$$

In the case where $r \gg s_\perp$, $a_2 - r \gg s_\perp$ (i.e., at a sufficient distance from the axis and walls of the bottle), formula (3.35) may be simplified by using asymptotic representations of the Bessel functions

$$n(r) \approx B\frac{e^{-\frac{r}{s_\perp}}}{(r)^{1/4}}. \tag{3.37}$$

The characteristic length $S_\perp$ is the effective length of transverse diffusion during the "recombination" lifetime of the particles.

4. Diffusion in a decaying plasma. In this section we shall consider the plasma decay occurring after the ionization sources have been turned off (for instance at the end of a pulsed discharge) in a cylindrical bottle whose axis is parallel to the magnetic field. The discussion concerns a later stage of the decay, when a constant temperature of the charged particles is being established that is close to the temperature of the surrounding medium, and the gas ionization is absent (z_i = 0).

a) Weakly ionized gas; dielectric bottle. The equation for the change in concentration taking place during the plasma decay is obtained from (3.5):

$$\frac{\partial n}{\partial t} - D_{a\parallel}\frac{\partial^2 n}{\partial z^2} - D_{a\perp}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) = 0. \tag{3.38}$$

A solution for this equation of type (3.8) which becomes zero at the walls of the cylindrical bottle (when z = 0, d, when r = a) may be represented in the form

$$n(r, z, t) = \sum_k A_k e^{-\left(\frac{D_{a\perp}}{\Lambda_k^2} + \frac{D_{a\parallel}\pi^2}{d^2}\right)t} J_0\left(\frac{r}{\Lambda_k}\right) \sin\frac{\pi z}{d}, \qquad (3.39)$$

$$\Lambda_k = \frac{a}{\varkappa_k}, \quad \varkappa_0 = 2{,}405, \quad \varkappa_1 = 5{,}520, \quad \varkappa_2 = 8{,}653, \ldots$$

Here χ_k are the roots of the Bessel function $J_0(\varkappa_k) = 0$, and k increases with the root.

The coefficients A_k are determined by the initial transverse distribution of concentrations (the longitudinal distribution is assumed to be sinusoidal). If the initial distribution is of the "diffusion" type (3.19), only the first term remains in the sum (3.39)

$$n(r, z, t) = n_0 e^{-\frac{t}{\tau}} J_0\left(\frac{r}{\Lambda_0}\right) \sin\frac{\pi z}{d}, \qquad (3.40)$$

$$\frac{1}{\tau} = \frac{1}{\tau_\perp} + \frac{1}{\tau_\parallel}, \quad \frac{1}{\tau_\perp} = \frac{D_{a\perp}}{\Lambda_0^2}, \quad \frac{1}{\tau_\parallel} = \frac{D_{a\parallel}\pi^2}{d^2}. \qquad (3.41)$$

The space distribution does not change with time, and the plasma decay obeys the exponential law. The decay time constant τ is determined by the diffusion coefficients. For an arbitrary initial transverse distribution of the concentration, the diffusion distribution (3.40) will be established in a period of time of the order of several τ after the start of this decay, since the decrease in time of the terms of series (3.39), which correspond to higher types of distributions (terms with $k > 0$), takes place faster than that of the basic type.

b) <u>Weakly ionized gas; metallic bottle</u>. The plasma decay in a metallic bottle is described by equations obtained from (3.12) and (3.13):

$$\left.\begin{aligned} \frac{\partial n}{\partial t} &= -D_{e\parallel}\left(1 + \frac{e}{T_e}\xi\right)\frac{\partial^2 n}{\partial z^2} - D_{e\perp}\left(1 + \frac{e}{T_e}\xi\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right), \\ \frac{\partial n}{\partial t} &= -D_{i\parallel}\left(1 - \frac{e}{T_i}\xi\right)\frac{\partial^2 n}{\partial z^2} - D_{i\perp}\left(1 - \frac{e}{T_i}\xi\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right). \end{aligned}\right\} \qquad (3.42)$$

A solution of this equation in the form of a diffusion distribution (3.40) and can be readily found [54]. Such a distribution is established over a sufficiently long time after the start of the plasma decay.

Substituting (3.40) into (3.42) and solving the system of equations with respect to τ and ξ (the quantity ξ according to (3.11), determines the potential of the metallic walls), we find

$$\tau = \frac{\tau_i + \frac{T_e}{T_i}\tau_e}{1+\frac{T_e}{T_i}}, \tag{3.43}$$

$$\frac{e\xi}{T_e} = \frac{\tau_e - \tau_i}{\tau_i + \frac{T_e}{T_i}\tau_e}. \tag{3.44}$$

The quantity τ_e, τ_i designates the effective time of the "free," (i.e., unchanged by the space charge) diffusion of electrons and ions:

$$\frac{1}{\tau_e} = \frac{D_{e\perp}}{\Lambda_0^2} + \frac{D_{e\parallel}\pi^2}{d^2}, \quad \frac{1}{\tau_i} = \frac{D_{i\perp}}{\Lambda_0^2} + \frac{D_{i\parallel}\pi^2}{d^2}. \tag{3.45}$$

The rate of the free diffusion of each type is equal, according to (3.45), to the sum of the rates of the longitudinal and transverse diffusion. Accordingly, the effective diffusion time (τ_e, τ_i) for each type of particle is found to be faster than either of these processes. The plasma decay constant is in turn determined by the greater of the quantities τ_e and τ_i (when $T_e \approx T_i$).

It should be noted that in the general case, the distributions of the electron and ion flows on the walls are not the same. Thus, when $D_{\parallel e}\frac{\pi^2}{d^2} \gg \frac{D_{\perp e}}{\Lambda_0^2}$, $\frac{D_{\parallel i}\pi^2}{d^2} \ll \frac{D_{\perp i}}{\Lambda_0^2}$ (this case occurs in a strong magnetic field when $d \gg a$), the electrons diffuse mainly along the magnetic field toward the end walls and the ions diffuse across the field toward the lateral surface of the bottle. The currents thus produced connect across the metallic walls.

We have considered the diffusion decay of a plasma bounded on all sides. In the case where only a part of the surface bounding the plasma is metallic (the ends or the lateral surfaces, for example), a solution of the decay equation becomes difficult.

c) <u>Transverse diffusion in a strongly ionized gas</u>. Plasma decay occurring as a result of diffusion across a strong magnetic field and recombination is described by an equation which can be obtained from (3.14):

$$\frac{\partial n}{\partial t} = D^0_{a\perp}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) + \gamma D^0_{a\perp}\frac{1}{r}\frac{\partial}{\partial r}\left(rn\frac{\partial n}{\partial r}\right) - \alpha n^2. \quad (3.46)$$

For an appreciable degree of ionization, when the diffusion caused by the electron-ion collisions and the recombinations are essential, the decay equation (3.46) becomes nonlinear. It then becomes convenient to characterize the plasma decay by the time constant defined by the average cross-sectional concentration:

$$\tau = \frac{1}{d\ln \bar{n}/dt}, \quad (3.47)$$

$$\bar{n} = \frac{2}{a^2}\int_0^a n(r)\,r\,dr. \quad (3.48)$$

Let us note that in the presence of nonlinear processes, τ is independent of the concentration, i.e., the plasma decay is not exponential.

Equation (3.46) cannot be solved in the general form. Reference [54] gives an approximate solution of the equation for a series of limiting cases which apparently describes a later stage of plasma decay. When equation (3.46) was solved to a first approximation, it was assumed that the right side of the equation was independent of the coordinates. The method of successive approximations was used to obtain a more accurate solution.

Fig. 7 shows the radial distribution of concentrations obtained from the approximate solution for several cases.

Curve 1 obtained for conditions where the nonlinear processes are nonessential $(\gamma n \ll 1,\ \alpha n \ll D^0_{a\perp}/a^2)$, provides a picture of the convergence of the method employed. The concentration distribution, determined to a second approximation, differs only slightly from the diffusion (3.19), which is an exact solution of the equation (broken curve).

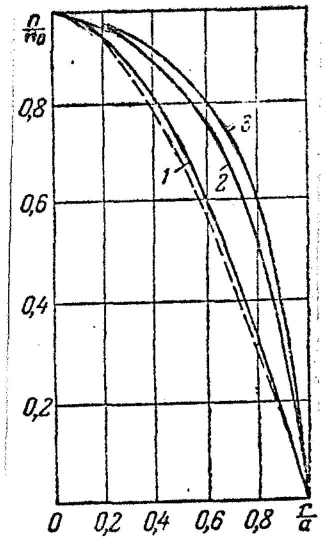


Fig. 7. Radial distribution of charged plasma particles.

Curve 2 represents the distribution of concentrations for the case where the main process of electron removal is the diffusion caused by electron-ion collisions $(\gamma n \gg 1,\ \alpha \ll \gamma D^0_{a\perp}/a^2)$. The plasma decay constant is then determined by the equality

$$\frac{1}{\tau_{(2)}} = 3{,}8\,\frac{\gamma D^0_{a\perp} n}{a^2} = 3{,}8\,\frac{\bar{D}_{ei}}{a^2}. \qquad (3.49)$$

Under conditions where the essential diffusion is due to electron-ion and electron-atom collisions, and the recombination may be disregarded, the concentration distribution in the process of plasma decay changes from curve 2 to curve 1. The reciprocal of the decay constant can then be approximated by the sum of the values pertaining to the limiting cases:

$$\frac{1}{\tau_{(2)}} = \frac{5{,}8}{a^2}\,D^0_{a\perp} + \frac{3{,}8}{a^2}\,\bar{D}_{ei} \qquad (3.50)$$

(the accuracy of this approximation is not worse than 20%).

At a high degree of ionization, when the diffusion due to electron-ion collision is nonessential $(\gamma n \gg 1)$ and the main processes of electron removal are the linear diffusion and the recombination, a radial distribution is established that is independent of time. As can be seen from the graph (curve 3), even when the influence of recombination is significant $\left(\frac{\alpha a^2}{\gamma D^0_{a\perp}}=4{,}5\right)$, the distribution is not very different from the nonlinear diffusion distribution (curve 2). The plasma decay constant at $\frac{\alpha a^2}{\gamma D^0_{a\perp}}<5$ can be approximated to within 10% by the formula

$$\frac{1}{\tau_{(2)}}=\frac{3{,}8}{a^2}\overline{D}_{ei}+1{,}2\alpha\bar{n}. \qquad (3.51)$$

In the case where linear diffusion and recombination are essential, the method described does not permit one to find a solution for equation (3.46). Reference [55] offers an approximate treatment of this case, taking into account the longitudinal diffusion based on the decay equation averaged over the volume. In averaging the equation it was assumed that the space distribution of the concentration is diffusional (formula (3.40), curve 1 in Fig. 7). Therefore, the solution is valid when the role of recombination is relatively minor. The plasma decay constant obtained from such a solution for conditions where the longitudinal diffusion is nonessential is found to be equal to

$$\frac{1}{\tau}=\frac{5{,}8}{a^2}D^0_{a\perp}+1{,}4\alpha\bar{n}. \qquad (3.52)$$

In the general case when all three of the processes of electron removal under consideration (linear and nonlinear diffusion recombination) are essential, the radial distribution of concentrations should obviously be represented by the curve situated between curves 1 and 3 in Fig. 7 (when $\frac{\alpha a^2}{\gamma D^0_{a\perp}}<5$). A correspondingly approximate expression for the reciprocal of the plasma decay constant may be obtained by combining formulas (3.49)-(3.51):

$$\frac{1}{\tau}=\frac{1}{\tau_0}+\frac{1}{\tau_{ei}}+\frac{1}{\tau_r}\quad\left(\text{when }\frac{1}{\tau_r}<1{,}5\frac{1}{\tau_{ei}}\right), \qquad (3.53)$$

$$\frac{1}{\tau_0}=\frac{5{,}8D^0_{a\perp}}{a^2},\quad \frac{1}{\tau_{ei}}=\frac{3{,}8\overline{D}_{ei}}{a^2},\quad \frac{1}{\tau_r}=1{,}2\alpha\bar{n}. \qquad (3.54)$$

II. EXPERIMENTAL INVESTIGATIONS OF THE DIFFUSION OF CHARGED PARTICLES OF A WEAKLY IONIZED GAS IN A MAGNETIC FIELD

4. Diffusion of Electrons in a Neutral Gas

Before evaluating the results of the experimental investigations of the diffusion of charged plasma particles across a magnetic field, we shall present the available experimental data on the transverse diffusion of electrons in a neutral gas.

The diffusional "dissolving" of an electron beam passing through a gas parallel to a magnetic field was studied in the early work of Bailey [56]. In this work it was found that in hydrogen at pressures of 2-16 mm Hg and magnetic fields less than 800 Oe, the transverse diffusion of electrons is adequately described by formula (1.11).

A more detailed study was undertaken by Bickerton [57]. The diagram of his instrument is shown in Fig. 8a. In this instrument, electrons released by an incandescent cathode C, moving in a homogeneous electric field, enter a cylindrical diffusion chamber through a small aperture O. A transverse diffusion of electrons takes place in the chamber, the magnetic field being directed parallel to the axis of the latter.

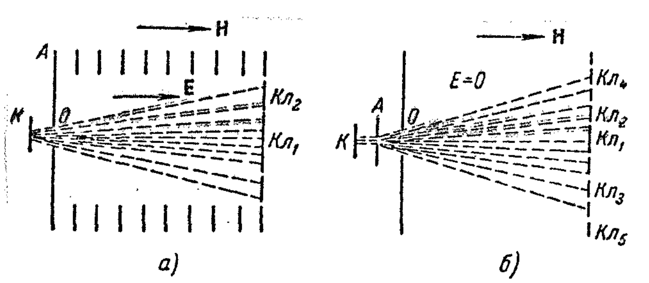


Fig. 8. Diagrams of experimental instruments.

K = C [cathode]
Кл = Cl [collector]

The increase in the transverse dimensions of the electron beam during the motion of the electrons from the entrance aperture to the collector is given by the order equality

$$\Delta r \approx \sqrt{4D_{e\perp}\tau} = \sqrt{4D_{e\perp}\frac{d}{u_{e\parallel}}} \tag{4.1}$$

(d is the length of the chamber and $u_{\| e}$ is the longitudinal electron velocity in the electric field).

The value of $u_{e\|}/D_{e\perp}$ can be found by measuring the ratio of the curves on the ring collector Kl_2 and the central collector Kl_1, which characterizes the expansion of the beam. Fig. 9 shows the results of a determination of this quantity in helium at small currents at which the space charge does not affect the motion of electrons. The data cited were obtained for conditions under which the electron emission from the collector, which alters the result of measurement, was absent.

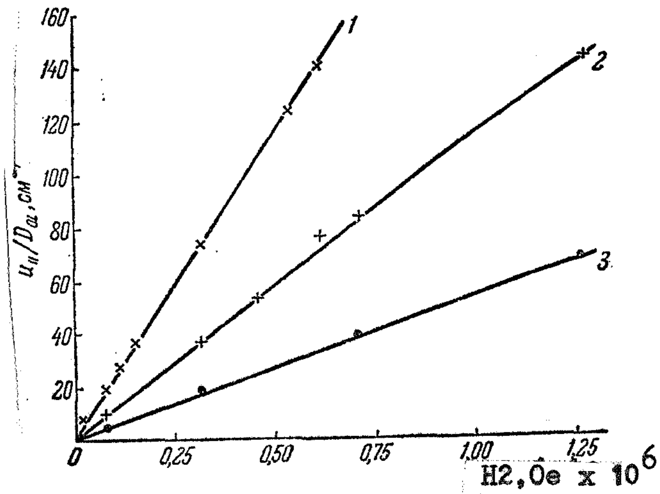


Fig. 9. The dependence of $u_{e\|}/D_{e\perp}$ on H.

$\frac{E}{p} = 10$ в/см mm Hg;
1 — $p = 0,5$ mm Hg;
2 — 1 mm Hg;
3 — 2 mm Hg.

The experimentally determined dependence of $u_{e\|}/D_{e\perp}$ in helium and hydrogen on the magnetic field $\left(\frac{u_{e\|}}{D_{e\perp}} \sim H^2\right)$ and on the pressure $\left(\frac{u_{e\|}}{D_{e\perp}} \sim \frac{1}{p}\right)$ is in good agreement with the theoretical dependence given by relations (1.10) and (1.11):

$$\frac{u_{e\|}}{D_{e\perp}} = \frac{3}{2}\frac{eE}{\bar{K}_e}\left(\frac{\omega_e}{\nu_{en}}\right)^2 = \frac{3}{2}\frac{\left(\frac{eE}{p}\right)}{\bar{K}_e}\left(\frac{p}{\nu_{en}}\right)^2\frac{e^2H^2}{m_e^2c^2p} \text{ when } \omega_e \gg \nu_{en}) \qquad (4.2)$$

($\bar{K}_e$ is the average electron energy).

The study of the transverse diffusion of electrons at lower pressures and over a wider range of magnetic fields was made by Zhilinskii, Terent'eva and the author. A diagram of the measurements is shown in Fig. 8b. According to this diagram, pre-accelerated electrons enter through the orifice into the diffusion chamber where there is no electric field. Once in the chamber, the electrons reach the collector by diffusing along the magnetic field. In the course of the longitudinal diffusion the electron beam expands to a size of the order of

$$r = r_0 + \Delta r, \quad \Delta r \approx \sqrt{4D_{e\perp}\tau} \approx \sqrt{2d^2 \frac{D_{e\perp}}{D_{e\parallel}}}. \qquad (4.3)$$

A measurement of the transverse distribution of current by means of a sectionalized collector makes it possible to find the ratio of the longidutinal to the transverse diffusion coefficients. The measurements were carried out in helium. The electron energy selected was less than 10 ev so that inelastic electron collisions would be absent. The pressure range was bounded at the lower end by the condition $\lambda_e \ll d$, which must be fulfilled in order that the motion of the electrons be of the diffusion type. The highest pressure was determined by the condition of small energy loss of electrons in the course of their diffusion $\left(\frac{m_e}{m_i}\frac{d^2}{D_{e\parallel}}\nu_{en} < 1\right)$.

Results of the measurements of the ratio of the diffusion coefficients for a helium pressure of 0.01-0.12 mm Hg and magnetic fields up to 2000 Oe are shown in Fig. 10. The same figure contains a theoretical curve obtained by means of equalities (1.10) and (1.11):

$$\frac{D_{e\parallel}}{D_{e\perp}} = \frac{\omega_e^2}{\nu_{en}^2} = \frac{e^2H^2}{m^2c^2\nu_{en}^2}. \qquad (4.4)$$

It was assumed in the calculations that $\frac{\nu_{en}}{p} = 2{,}5\cdot 10^9$ 1/sec.mm Hg (the frequency of collisions between electrons and atoms of helium at energies of 2-15 ev is approximately constant). It is apparent from the graph that the experimental results practically coincide with the theoretical values; not only the dependence of $D_{e\parallel}/D_{e\perp}$ on H and p, but the absolute values of this quantity agree as well.

Thus, the experimental data show that the diffusion of electrons across a magnetic field in a neutral gas (in the absence of transverse

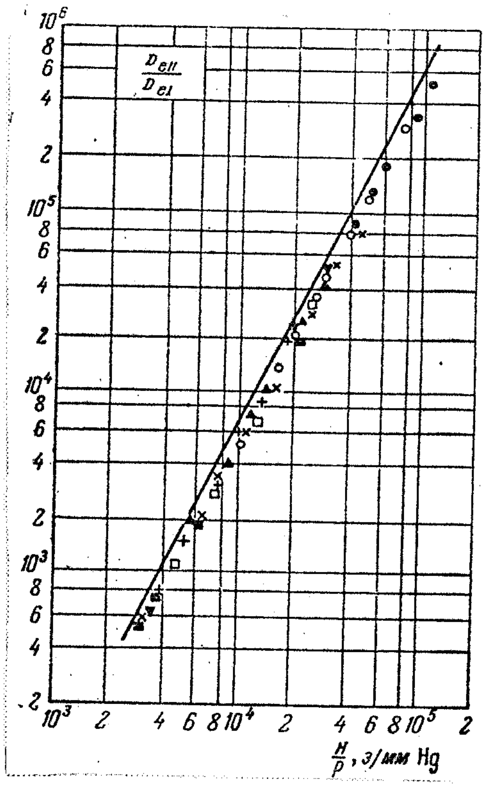


Fig. 10. $D_{e\parallel}/D_{e\perp}$ versus H.

Experimental points:

● - p = 0.01 mm Hg;
○ - 0.02 mm Hg;
▲ - 0.035 mm Hg;
× - 0.06 mm Hg;
+ - 0.075 mm Hg;
□ - 0.085 mm Hg;
■ - 0.12 mm Hg;
▬ - theoretical curve.

fields and for a slight effect of the space charge) is adequately described by the known theoretical formulas. Let us note that in the presence of a transverse electric field and an essential influence of the space charge, the transverse motion of the electrons becomes somewhat more complicated. Thus, at low gas pressures, an instability of the motion of the electrons and intense oscillations and noises have been detected in static magnetrons (see, for example, reference [58]).

5. Diffusion of Charged Particles from the Plasma of a Discharge with an Incandescent Cathode.

The first experimental data on the diffusion of charged particles in a magnetic field were obtained from a study of ion sources published in 1949 [59]. A diagram of the apparatus described in reference [59] is given in Fig. 11. A stationary discharge between the incandescent cathode K and anode A is maintained at a low gas pressure (10^{-4}--10^{-2} mm Hg). The plasma along the lines of force of the magnetic field penetrates through a diaphragm into the cavity of the anode assembly. The size and shape of the region of the "primary" plasma in cavity I depends on the size and shape of the anode diaphragm (in the experiments the diaphragm was a narrow slit and the anode cavity was in the shape of a rectangular parallelepiped). Charged particles of the primary plasma diffused toward the periphery of the anode cavity.

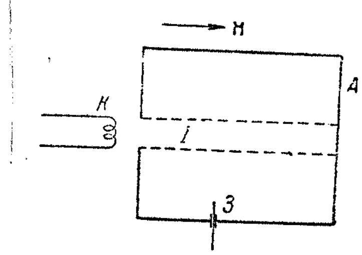


Fig. 11. Diagram of the experimental apparatus.

By measuring the concentration distribution of the charged particles in the plane perpendicular to the magnetic field, it is possible, as was shown in 3 of Chapter I, to obtain data on the coefficient of transverse diffusion. The measurement of the concentration distribution was accomplished by means of a mobile probe (10^{-4}--10^{-2} mm Hg). In order to reduce the influence of the magnetic field on the sounding measurements, the plasma concentration was determined from the ion current at the probe (the probe was kept at a potential which was negative with respect to the plasma).

The measurements determine the characteristic length of the concentration drop $S_{\perp}$ for several conditions in argon and hydrogen at a pressure of $\sim 10^{-3}$ mm Hg and a magnetic field of 3,000-4,000 Oe. Bohm compared the experimental values of $S_{\perp}$ with the results of calculations based on the assumption of a dipolar character of the particle motion, and came to the conclusion that the rate of the transverse diffusion is approximately 2 orders of magnitude greater than the rate of dipolar diffusion across the magnetic field [59]. The discrepancy was explained in the work of Simon [47, 48, 7]. Simon showed that owing to the equalization of the potential at the plasma boundaries, the diffusion in the volume bounded by the conducting walls is not dipolar. As was shown in 3 of chapter I, the conducting walls should be charged to a negative potential of the order of $\frac{kT_e}{e}\ln\frac{n}{n_{\text{rp}}}$, so as to make the flow of electrons in the region of diffusion zero (i.e., the conducting cavity becomes a potential box for the electrons).

This is confirmed by measurements of the anodic current distribution described in the paper of Zharinov [51]. The measurements were made by means of a probe moved in the vicinity of the end surface of the anode (a cylindrical instrument similar to the one shown in Fig. 11 was used). It was found, in accordance with the above, that outside the region of primary plasma the electron flow to the anode was substantially smaller than the ion flow.

When the flow of electrons is equal to zero, the distribution of the plasma concentration is determined by the diffusion of the ions. The characteristic length $S_\perp$ is given, in accordance with (3.30), by the distance to which the ions are able to diffuse during their lifetime*:

$$s_\perp = \sqrt{D_{i\perp}\tau_{i\|}}, \quad \tau_{i\|} = \frac{d^2}{\pi^2 D_{i\|}}. \tag{5.1}$$

*Let us note that the formulas used in [47, 48] differ from (3.30) by the factor $\sqrt{2}$. This difference is due to the fact that Simon neglected the transverse electric field in his calculation (see p. 48).

Substituting expression (1.10) and (1.11) for the diffusion coefficients of ions and assuming $\omega \gg \nu_{in}$, we find

$$s_{\perp} = \frac{d}{\pi} \frac{\nu_{in}}{\omega_i}. \tag{5.2}$$

This relation applies if the longitudinal motion of the ions is of the diffusion type, i.e., if $\lambda_i \ll d$. When $\lambda_i > d$, the lifetime of the ions is determined by their thermal velocity

$$\tau_{i\parallel} \approx \frac{d\sqrt{2m_i}}{\sqrt{T}} \quad (\text{when } T_i \approx T_e), \tag{5.3}$$

and the quantity S is given by the order relation

$$s_{\perp} \approx \left(\frac{2T}{m_i}\right)^{1/4} \frac{(\nu_{in}d)^{1/2}}{\omega_i}. \tag{5.4}$$

To verify Simon's postulations, measurements were undertaken of the dependence of $S_{\perp}$ on the magnetic field [48, 7]. The measurements were carried out in a cylindrical anode assembly made of metal as in the diagram shown above (Fig. 11). Most of the results were obtained in a chamber filled with nitrogen at a pressure of 10^{-3}--5×10^{-3} mm Hg and a magnetic field of 2,000--14,000 Oe. The degree of ionization in the primary plasma was less than 1%. A typical dependence of $S_{\perp}$ on H is shown in Fig. 12. As can be seen from (5.1), this dependence is in agreement with the theoretical one and corresponds to the quadratic dependence of the transverse diffusion coefficient on the magnetic field $\left(D_{\perp} \sim \frac{1}{H^2}\right)$. Measurements of the dependence of the length $S_{\perp}$ on the nitrogen pressure were also undertaken. When $\lambda_i \ll d$ ($\lambda_i < 6$ см, $d = 26$ см), a linear increase of $S_{\perp}$ with the pressure was obtained which agreed with (5.2); when $\lambda_i \approx d$ ($\lambda_i \approx d = 6$ см) it was found that $s_i \sim \sqrt{p}$ (see 5.4)).

The absolute values of $S_{\perp}$ obtained in the experiments described in [48] as well as in the first experiment [59], agree in order of magnitude with the theoretical values. It would be difficult to expect

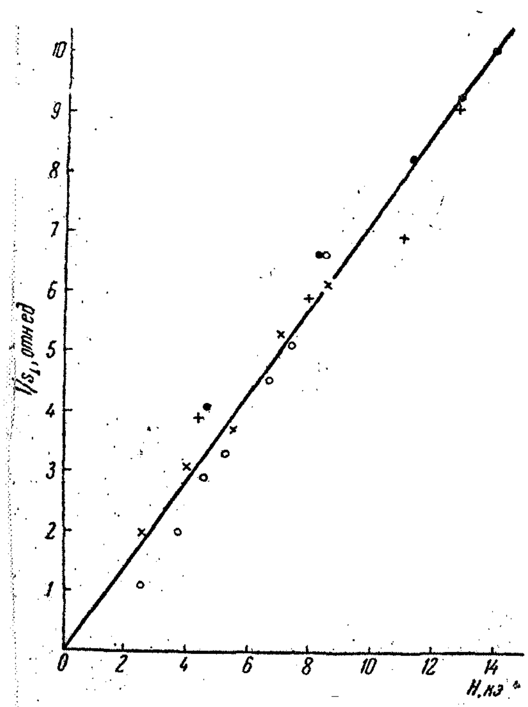


Fig. 12. $1/s_{\perp}$ versus H.

a better agreement, since the effective collision cross section and the ion temperature were known only very approximately.

Results of investigations of the concentration distribution of charged particles diffusing from a stationary plasma are also given in reference [60]. The difference in these investigations is that the diffusion was studied in a bottle of greater length (1.2--1.5 m) and a substantial part of the lateral surface of the bottle was made of glass. A rigorous analysis of the diffusion is difficult for such a "mixed" surface of the bottle. It seems probable, however, that when $s_{\perp} \ll a$ the role of the boundary conditions on the lateral walls is minor, and the measurements of the length of the drop in the diffusion concentration may be compared to the relations (5.2) and (5.4).

The measurements carried out in nitrogen and in hydrogen at a pressure of 10^{-3} mm Hg have shown that the length $S_{\perp}$ is inversely proportional to the magnetic field strength H (in the range up to 3,500 Oe). The order of magnitude of $S_\perp$ agrees with that obtained from formula (5.2). The quantity $S_\perp$ was also determined in H_2, He, N_2, Ne, Ar, Kr at $p = 10^{-5}$ mm Hg. Measurements at low pressure, however, are difficult to interpret since $S_\perp$ is of the order of the Larmor ionic radius and the standard theory is inapplicable.

Thus the experimental data obtained to date for the distribution of concentrations in a plasma diffusing across a magnetic field from a discharge with an incandescent cathode are in good agreement with the "collisional" diffusion theory.

It should be noted that no explanation can be obtained on the transverse motion of electrons from these data, since under the conditions of these experiments (in a bottle with conducting walls) the distribution of concentrations is completely determined by the diffusion of ions [51].

In many experiments there were observed some anomalous phenomena in the plasma of a discharge with an incandescent cathode; these phenomena may be associated with a change in the character of the transverse diffusion of the charged particles in the magnetic field.

As early as the first experiments described in Bohm's article [59], it was found that the ratio of the electron saturation current at the probe to the ion current in a strong magnetic field is considerably greater than the expected value (in these experiments the collecting surface of the probe was perpendicular to the magnetic field). This led to the assumption that the transverse motion of electrons occurs faster than the diffusion. Reasons for this belief were rather few since the examination of the electron branches of the sounding curves in the magnetic field were very approximate.

Further studies of the sounding characteristics in a plasma of a discharge with an incandescent cathode were carried out by Zharinov [61, 62]. In these investigations, holes were drilled in the face of the anode, and the volt-ampere characteristics of the current were measured with a probe located behind the hole. As the magnetic field rose to some critical value, the ratio of the electron saturation current to the monotonic ion current decreased. At the "critical" magnetic field, this ratio abruptly increased and at the same time intense oscillations of the probe current were observed. Systematic measurements of the critical magnetic field were performed in hydrogen, nitrogen and helium in the pressure range of 10^{-2}--10^{-3} mm Hg. It was found that the critical field rises in an approximately linear fashion with the pressure, and that at a pressure of $\sim 5 \times 10^{-2}$ mm Hg, it amounts to approximately 500 Oe for H_2 and about 1,500 Oe for N_2 and He.

By comparing the currents to the different probes (the phases of current oscillation were compared) it was established that at fields greater than the critical, there are formed one or two plasmoids (tongues) which revolve around the axis of the discharge. As the magnetic field increased, the period of revolution of the tongue decreased. Similar phenomena are described in ref. [63]. The authors of this work observed the revolution of the plasmoid in various gases, H_2, He, N_2, Ar, Kr, Xe, at pressures in the anode cavity between 10^{-4}--10^{-5} mm Hg (the pressure in the gap between the cathode and the anode was kept at about 10^{-3} mm Hg. At pressures of 10^{-3} mm Hg in the anode cavity, the revolving plasmoid was not observed in reference [63]. In order to study the formation and revolution of plasmoids in more detail, Elizarov and Zharinov worked out a special technique [64]. In these experiments, the end surface of the anode consisted of a metallic grid. The flux of electrons or ions penetrating through the grid was transformed into a flux of light. The electrons were accelerated by short voltage pulses and, impinging on a fluorescent screen, caused it to glow. Under conditions of an "ionic image," the ions were accelerated and knocked secondary electrons out of the grid; after being accelerated, there electrons were directed toward the phosphor. In these measurements, the discharge was produced by pulses 1 msec long. By photographing the glow of the fluorescent screen for various delays of the "sounding pulses" relative to the start of the discharge, it was possible to record the distribution of the plasma concentrations over the cross section of the anode during the period of formation and revolution of the plasmoids. Preliminary results of investigations published thus far indicate that the shapes of the plasma formations and their motions are quite varied.

It should be noted that the result of the various investigations of the "anomalous" discharge conditions involving an incandescent cathode, described in references [61--64], are contradictory in many respects. The mechanism of formation of the revolving plasmoids is not understood. The relationship between the effects observed and the transport phenomena in the plasma has not been identified.

In a recent paper [65], Guest and Simon attempted to relate the observed anomalies with the appearance of a corkscrew plasma instability similar to the instability of the positive column of a discharge, earlier examined by Nedospasov and Kadomtsev (p. 73). The corkscrew instability may be caused by the longitudinal current in the region of diffusion (as was indicated above, in diffusion in a metallic container, the ion current on the periphery is not balanced by the electron current).

It is not clear as yet whether the anomalies occurring in the diffusion of a plasma from a discharge with an incandescent cathode can be explained in this fashion, since a detailed comparison of the conditions under which the instability arises with the experimental data has not been made to date.

6. Diffusion in a Low Pressure Gas Discharge

Under many conditions of a low-voltage arc discharge (at gas pressures of 0.1--10 mm Hg), the ionization of the gas occurs mainly in the vicinity of the cathode (see [66, 67]). The distribution of the plasma concentrations in the space between the cathode and the anode is determined by the diffusion of the charged particles from the region adjacent to the cathode. If we assume that the temperature of the charged particles is constant over the entire volume (outside the region of ionization), we can apply to this case the results of the analysis performed in No. 3 of chapter I. It follows from this analysis that at some distance from the cathode the plasma concentration should decrease exponentially in the direction of the anode. The characteristic length of the concentration drop in a cylindrical discharge bottle (made of a dielectric) along the axis of which is directed the magnetic field, is given by the relation (3.33):

$$s_{\parallel} = \frac{a}{2{,}405}\sqrt{\frac{D_{a\parallel}}{D_{a\perp}}}. \qquad (6.1)$$

Nedospasov's work [68] gives experimental data on the distribution of the plasma concentrations between the cathode and the anode of a low-voltage arc discharge in argon placed in a longitudinal magnetic field. These data were obtained by measuring the ion currents on the wall probe which was moved along the lateral wall of the discharge bottle. The measurements determined the characteristic length $S_{\parallel}$ and the latter was used to find the ratio of the coefficients of the longitudinal to the transverse diffusion $D_{a\parallel}/D_{a\perp}$ (see (6.1)). The variation of the ratio $D_{a\parallel}/D_{a\perp}$ with the magnetic field strength is shown in Fig. 13. As can be seen from the graph, the experimental dependence at magnetic fields below 1,000 Oe and argon pressures of 0.25--1 mm Hg agrees well with the theoretical dependence resulting from formulas (1.23) and (1.24):

$$\frac{D_{a\parallel}}{D_{a\perp}} = 1 + \frac{\omega_e \omega_i}{\nu_{en}\nu_{in}} = 1 + \frac{e^2 H^2}{c^2 m_e m_i \nu_{en} \nu_{in}} . \quad (6.2)$$

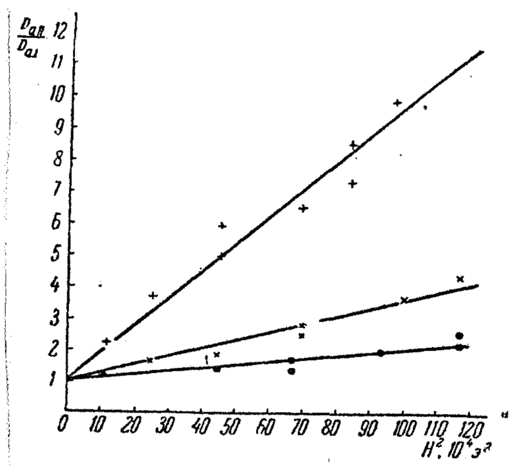


Fig. 13. $D_{a\parallel}/D_{a\perp}$ versus H.

+ - p = 0.25 mm Hg;
✕ - 0.7 mm Hg;
● - 1.0 mm Hg.

[$э^2$ = Oe^2]

7. Diffusion of a Positive Discharge Column in a Plasma

It has been stated above (see p. 44), that the main characteristics of a positive discharge column -- the electron temperature, the distribution and maximum concentration of the plasma, the longitudinal electric field strength -- are determined by the diffusion of the charged particles toward the lateral walls of the discharge bottle. Therefore, by studying these characteristics in the presence of a magnetic field directed parallel to the axis of the discharge, one can obtain information on the transverse diffusion of the charged particles.

Sounding measurements of the characteristics of the positive discharge column in helium at low magnetic fields (up to 400 Oe) have been carried out by Bickerton and Engel [69]. The results of the measurements are in good agreement with the diffusion theory of a positive discharge column. As the magnetic field increases, the electron temperature and the longitudinal electric field strength decrease, and the plasma concentration increases. These results are also confirmed by other studies. In the work of Vasil'eva and Granovskii [70] the diffusion coefficient was measured directly in the positive column. The plasma flow toward the lateral wall and the concentration gradient in the vicinity of the wall were determined by means of probes (from the ionic parts of the characteristics). The ratio of these values is equal to the diffusion coefficient. The change in the coefficient of transverse diffusion in helium corresponds to formula (1.24) when the magnetic field increases to 500--1,000 Oe (at pressures of 0.07--1 mm Hg).

However, the condition of the positive discharge column depends on the diffusion due to particle collisions only when the magnetic fields are not too great. As the magnetic field increases, starting at some "critical" value, the rate of escape of the particles also increases. At the same time, intense oscillations and noises appear. This effect was first noted by Lehnert [71] and was studied in detail by Lehnert and Hoh [72-74], Granovskii et al [75], Allen et al [76, 77] and in a number of other studies [78-80]. We shall cite here some experimental results.

In references [71-74] and [76-77], the investigations of the positive column were carried out in a discharge tube of considerable length

(2.4--4.1 m), placed in a homogeneous magnetic field. The cathode and anode were outside the boundaries of the magnetic field. Measurements were made of the potential difference between identical probes placed in the plasma at some distance from each other, and from this value the longitudinal electric field strength was obtained. The results of the measurements in helium for a bottle radius of 1 cm are shown in Fig. 14 by means of continuous curves. In the same figure, the broken lines represent the theoretical curves calculated with the assumption that the diffusion is determined by collisions.*

At small magnetic field strengths, the shape of the theoretical and experimental curves is the same. Some differences are due to the inaccurate values of the effective collision cross sections used in the calculations. Beginning with the critical magnetic field H_{cr}, the magnitude of the longitudinal electric field strength in the positive column increases. This indicates an increase in the rate of escape of the charged particles from the plasma. The increase in the rate of escape of the charged particles at magnetic fields greater than critical has also been confirmed by measurements of the ion current on the wall probes [74], [75].

At higher magnetic fields (4,000--6,000 Oe), the longitudinal field strength (and correspondingly the rate of escape of the particles from the plasma) is found to be of the same order as in the absence of the magnetic field and undergoes little change.

The increase in the longitudinal electric field strength at $H > H_{cr}$ is accompanied by a sharp increase in the noise intensity. The values

*These theoretical curves have been plotted for the case where the plasma contains only atomic He^+ ions. The presence of molecular ions does not substantially affect the results of the calculations.

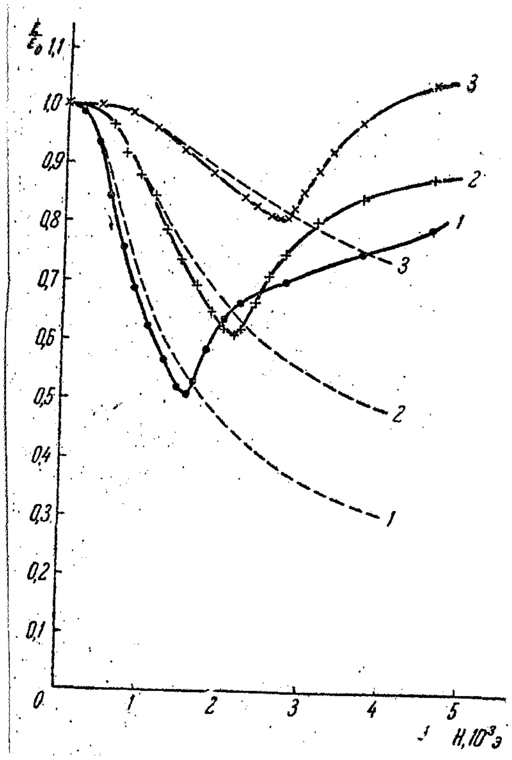


Fig. 14. Longitudinal electric field strength versus magnetic field strength.

1—p = 0.45 mm Hg;
2—0.9 mm Hg;
3—1.8 mm Hg.

[э = Oe]

of the critical magnetic fields determined from the minima of the E (H) curves and from the increase of the noise intensity agree with each other.

Fig. 15 shows the dependence of H_{cr} on the pressure of helium for several values of the diameter of the discharge bottle as given in references [73, 76]. It was found that the value of H_{cr} increases slightly when the discharge current is increased and changes correspondingly little when the length of the discharge region located in the magnetic field is decreased from 4 to 0.5 m. We are citing here only the results for helium. Similar data were obtained in studies of the positive discharge column in hydrogen, nitrogen, neon, argon and krypton [73, 74, 76, 77].

The abnormally fast escape of particles from the plasma of a positive discharge column was explained in the paper of Kadomtsev and Nedospasov [81] who showed that this escape may be due to the appearance of an instability of the positive column in the magnetic field.

[$H_{кр} = H_{cr}$]

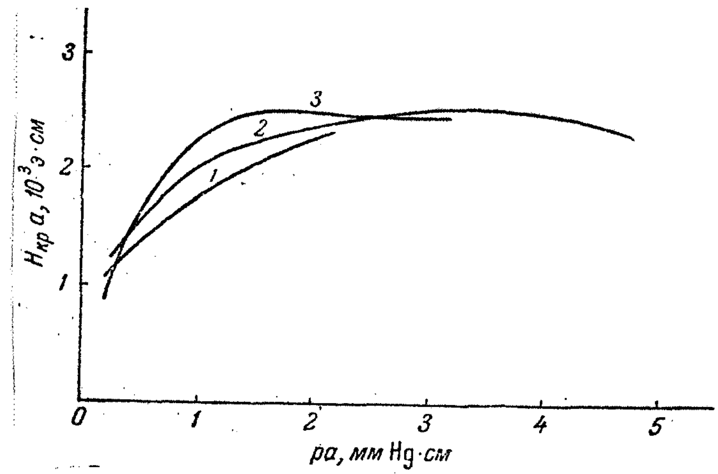


Fig. 15. Dependence of H_{cr} a on pa.
1 - Curve calculated from the formulas of reference [81];
2 - Experimental curve from the data of [76]; a = 0.9-1.3 cm;
3 - Experimental curve from the data of reference [73]; a = 0.75 cm.

The paper discusses a perturbation consisting of a serpentine (corkscrew) distortion of the plasma column.* The increase in perturbation is hindered by the diffusion since the diffusion flow of particles to the wall is greater where the column is closer to the wall. At the same time, a right-hand corkscrew distortion is associated with a force directed toward the walls and acting on the column in the magnetic field. This force is associated with the azimuthal component of the current arising from the distortion. As the magnetic field increases, this force, tending to increase the distortion, also increases, and the diffusion

*Let us note that to explain the observed anomalies Hoh [82] developed a rather unclear hypothesis on the instability of the layer between the plasma and the wall (instability not associated with the discharge current). This hypothesis cannot account for the phenomena observed. Judging from the latest articles [83, 84] the author himself has repudiated this hypothesis and maintains that the corkscrew instability discussed by Nedospasov and Kadomtsev arises in the positive column.

inhibiting the perturbation slows down. For this reason, starting from a certain critical value of the magnetic field, an instability is produced. The values of H_{cr} for this instability (see Fig. 15), calculated from the formulas given in reference [81], are in good agreement with the results of the experimental determination of this value in the positive column of a discharge.*

The onset of a helical instability in the positive column of a discharge, predicted by Kadomtsev and Nedospasov, was observed directly at magnetic fields exceeding the critical value in references [76, 77]. The authors of these papers were able to photograph the time resolution of the glow of a plasma column in two projections under certain conditions. The photographs obtained constitute clear-cut evidence for a right-hand distortion of the plasma column. An additional confirmation of this theory came from measurements of H_{cr} in the positive column of a discharge in the presence of a high-frequency field [80]. A field with a frequency of 4 MC was supplied from the generator to a special solenoid in which the discharge bottle was placed. In the presence of the additional ionization produced by the high-frequency electric field, the constant electric field necessary for maintaining the equilibrium of the particles in the positive column decreases. The concentration of electrons for the given discharge current correspondingly increases. Therefore, when the plasma column is distorted, the stabilizing effect of diffusion is reinforced and the magnitude of H_{cr} should increase. Experiments have confirmed the postulated increase in H_{cr}.

*The theoretical curve of Fig. 15 was plotted from the data kindly supplied to the author by A. V. Nedospasov. The same data were used in the paper by Vdovin and Nedosposov [119] published after this survey had been written. Reference [119] gives the results of a calculation of H_{cr} for helium, neon, argon and mercury, and the respective experimental data are analyzed.

Reference [81] not only derives a criterion for the onset of helical instability, but also discusses the development of instability at values of H close to H_{cr}. This analysis determined the frequencies of oscillation of the plasma column and the rate of the anomalous diffusion associated with these oscillations. These results are in accord with the experimental results.

At fields considerably higher than H_{cr}, many types of oscillations may be observed in the column simultaneously; these oscillations should then become irregular. The theory of such a developed instability was formulated by Kadomtsev [85] by analogy with the turbulent convection of a fluid. It follows from this theory that the rate of the "turbulent" diffusion of a plasma at $H \gg H_{cr}$ should not depend on the magnetic field. These as well as other conclusions of the theory agree with the experimental results.

Thus it may be considered certain that the anomalous increase in the rate of escape of the charged particles from the positive column of a discharge in a strong magnetic field as well as the observed oscillations and noises are due to the onset and development of a helical instability of the plasma column.

8. Diffusion in a Discharge with Oscillating Electrons

A diagram of a discharge with oscillating electrons (a Penning discharge) is shown in Fig. 16. In this type of discharge, the electrons oscillate between the cathode and the reflector (which are connected electrically) until they reach the anode A as a result of displacement across the magnetic field. The ratio of the plasma concentration on the periphery of the discharge (in the vicinity of the lines of force of the magnetic field intersecting the anode) to the concentration in the central region n_a/n_o may serve as a measure for determining the rate of diffusion of the plasma across the magnetic field. This ratio was determined in the work of French investigators for various experimental conditions [86-88]. The plasma concentration was measured by means of probes (from the ion current to the probes).

The results of measurement n_a/n_o for a Penning discharge with a cold cathode in hydrogen are shown in Fig. 17. At small magnetic fields, the ratio h_a/n_o decreases monotonically with a rise in the magnetic field and increases with the pressure. Such a variation correlates with the variation in the rate of diffusion caused by the collisions. However, starting at the critical field H_{cr}, the ratio n_a/n_o increases, while in a "supercritical" regime, n_a/n_o decreases as the pressure increases. The value of H_{cr} rose with an increase in pressure and a decrease in the radius of the discharge, and remained practically unchanged as the discharge current changed by a factor of 10. At fields somewhat in excess of the critical value, intense noises were observed. In addition to the low frequency oscillations, noises were detected by means of an external detector at frequencies of about 1,000 MC. The change in the amplitude of the noises corresponded qualitatively to the change in the value n_a/h_o. Similar effects were observed in a Penning discharge with an incandescent cathode for considerable dimensions of the apparatus (a length of about 1 m). The anomalous character of the change in n_a/n_o and the appearance

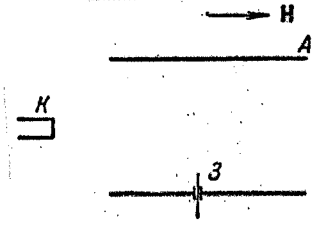
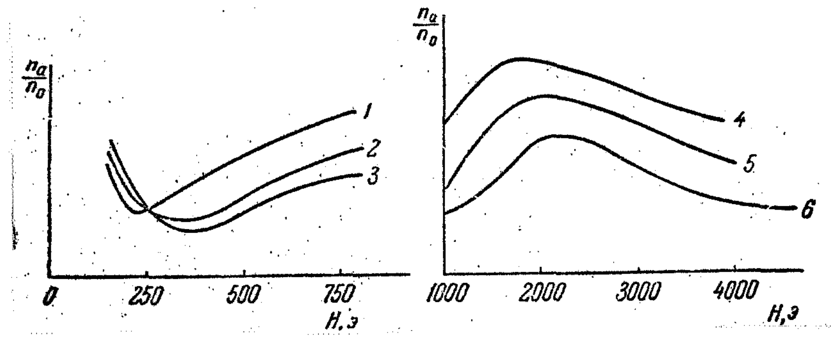


Fig. 16. Diagram of a discharge with oscillating electrons.

[K = C]
[3 = P]

of intense noises in magnetic fields greater than the critical, obviously indicate the onset of an unstable state in the Penning discharge, characterized by an accelerated transverse transport of particles. The nature of the observed anomalies is unclear.



[H, Э = H, Oe]

Fig. 17. Dependence of n_a/n_o on H.
1 - p + 0.016 mm Hg; 2 - 0.024 mm Hg; 3 - 0.03 mm Hg;
4 - 0.021 mm Hg; 5 - 0.025 mm Hg; 6 - 0.03 mm Hg.

References [87, 88] indicate that in a Penning discharge, instabilities are possible which are due to the anisotropy of the velocity distribution function of electrons and to the electric fields in the plasma. However, a concrete analysis of the mechanism of these instabilities is lacking.

9. Diffusion in a Decaying Plasma

We have presented above the results of a series of experiments in which an anomalously fast motion of the charged particles across the magnetic field was observed. In the majority of cases, the anomalies observed are attributed to the presence of a current through the plasma or to a strong directed motion of the charged particles. For this reason, in checking the diffusion theory, it is particularly interesting to consider the studies of the diffusion of a decaying plasma which can be conducted under conditions where the state of the plasma is close to one of equilibrium.

The first experiments in the study of plasma decay in a magnetic field were carried out by Bostik and Levine in a toroidal chamber [89]. In such geometry, the magnetic field is inhomogeneous and the diffusion of the plasma is masked to a considerable extent by the toroidal drift. Nevertheless, we shall dwell on the results of reference [89], since they are often used in an analysis of the transverse diffusion of a plasma.

The plasma was studied in a metallic toroidal vacuum chamber of rectangular cross-section. This chamber functioned simultaneously as a resonator excited at two spaced frequencies in the 10 cm range. Under the influence of the high frequency power, an impulsive discharge in the gas filling the chamber was achieved at one of the resonance frequencies. The shift of the resonance frequency corresponding to the second resonance was measured by means of a measuring generator during the period between the pulses. The frequency shift is proportional to the mean concentration of the plasma under certain conditions (see for example reference [90]). The change in the concentration of the decaying plasma and the time constant of plasma decay τ were thus determined. The value of τ in helium was approximately linear and increased when the pressure was raised from 0.05 to 0.35 mm Hg. When the magnetic field was changed from 160 to 1400 Oe, the measured time constant had a maximum. The presence of a maximum has been considered by many as evidence of the "anomalous" transverse diffusion.* This interpretation is not convincing <u>in our view,</u> since it does not allow for the effect of the toroidal drift

*The oscillations associated with plasma decay detected in reference [89] are sometimes considered as another indication of the anomalous phenomena. However, reference [89] does not explain under what conditions the oscillations were observed. In particular, it is not clear whether the current through the plasma during the period of oscillations is completely absent. No correlation has been established between the observed oscillations and the rate of plasma decay.

caused by the inhomogeneity of the magnetic field on the plasma decay. Furthermore, the relatively weak dependence of the rate of plasma decay on the magnetic field was established in reference [89] (an 8--10-fold increase in H produces a 1.5--2-fold increase in τ) as well as a decrease in the rate of decay associated with an increase in pressure, is characteristic of a toroidal drift of charged particles in a neutral gas. Therefore, the results obtained in reference [89] obviously cannot be explained by the toroidal drift of the plasma. In any event, the data of this work can provide very little information of the transverse diffusion at magnetic fields greater than 200--300 Oe, since the rate of diffusion is found to be less than the rate of the toroidal drift.

Investigations of plasma decay in a homogeneous magnetic field were carried out by using the sounding method and the microwave method. In the experiments of Granovskii and coworkers [91--93], the plasma decay was studied by means of probes. The plasma was generated in cylindrical glass bottles 2--3 cm in diameter by means of pulses of a high-frequency discharge or a discharge with an incandescent cathode. The change in the ion current to the probes (I_a), introduced into the bottle were recorded during the interval between the pulses. The slope of the curves $\ln I_a = f(t)$ was used to determine the time constant of the plasma decay in the initial stage τ_0 (immediately after the discharge pulse) and at a later stage τ_1 (apparently, a few hundred microseconds after the pulse). The measurements were carried out at plasma concentrations greater than 10^9--10^{10} cm^{-3}.

The dependence of τ on the magnetic field in helium and argon, obtained in references [91--93] for several cases, is shown in Fig. 18. At magnetic fields greater than 500--1,000 Oe, the time constants undergo little change. The saturation of the curves may be due to the bulk processes of removal of charged particles, such as the electron-ion recombination and the capture of electrons by the molecules of electronegative impurities with subsequent ionic recombination. To verify the influence of bulk processes, a determination was made in reference [92] of the relation between the total number of particles escaping to the surface of the bottle and measured by the ion currents to the wall probes, and the initial number of charged particles estimated from the ionic currents to the probes placed inside the volume of the bottle.

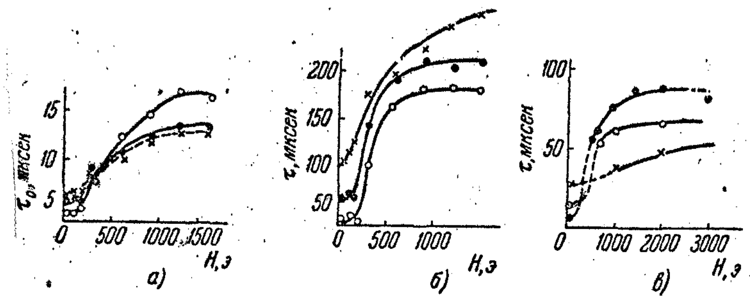


Fig. 18. Dependence of τ on H.

а) He, τ_0; ○ — p = 0,06 мм рт. ст.; × — 0,33 мм рт. ст.; ⊗ — 0,11 мм рт. ст.
б) He, τ_1; ○ — p = 0,06 мм рт. ст.; × — 0,33 мм рт. ст.; ⊗ — 0,11 мм рт. ст.
в) Ar, τ_0; ⊗ — p = 0,03 мм рт. ст.; ○ — 0,1 мм рт. ст.; × — 0,3 мм рт. ст.

[рт. ст. = Hg]

The results of these measurements clearly show the essential role of the bulk processes of removal of charged particles associated with the decay of the helium plasma under the conditions of the experiments described. Thus, the curves $\tau(H)$ are determined by diffusion of the plasma only at small magnetic fields (from 500--1,000 Oe). Qualitatively, the course of the curves in this region agrees with the diffusion theory. However, a quantitative comparison is difficult because the plasma parameters have not been fully determined. Sounding measurements in a magnetic field determine only the order of magnitude of the plasma concentration. During the initial stages of plasma decay, the electron and the ion temperature are not known either. A suitable selection of the parameters can insure agreement between the experimental and the calculated values of τ.

It follows from the above that the results of the experiments described in references [91--93] do not contradict the diffusion theory, but these results are insufficient for a qualitative verification of the theory.

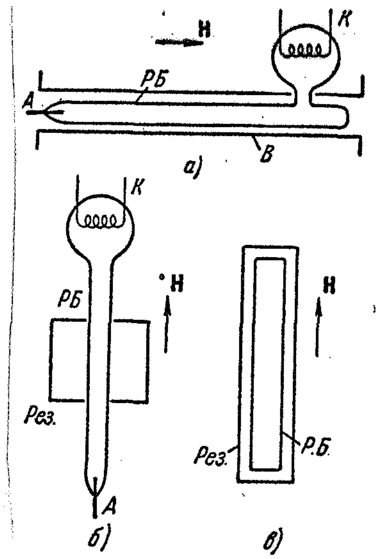
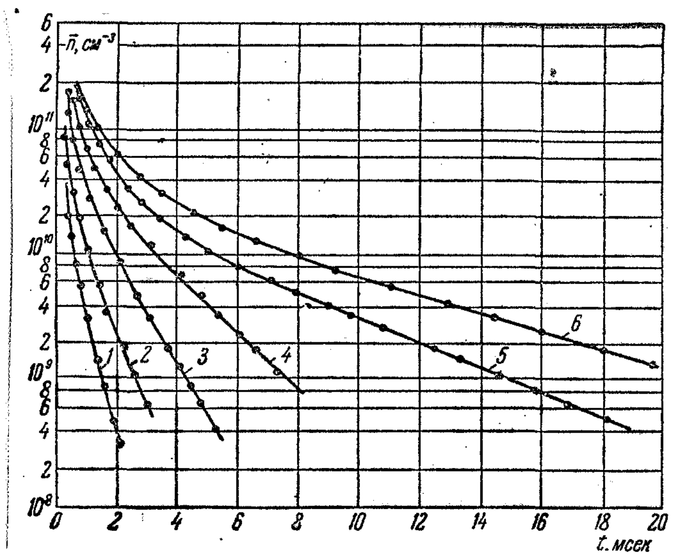


Fig. 19. Diagram of the introduction of plasma into measuring waveguides and resonators.
K —cathode; A —anode; РБ —discharge bottle; B —waveguide; Рез. —cavity resonator.

The use of microwave methods of investigation [90] has made it possible to study the plasma decay of concentrations from 10^7--10^8 to 10^{12}--10^{13} cm^{-3} (the decay in helium was studied). In this section, we shall examine the results of investigations conducted at small concentrations at which the collisions of the charged particles with one another are not essential.

In the work of Zhilinskii and the author [94--95], the plasma decay in a magnetic field was studied by the waveguide method. A cylindrical glass bottle with an inner diameter of 1.6--2.0 cm and 110 cm long was placed in a cylindrical waveguide (Fig. 19a). The plasma was generated in helium by pulses of a current discharge 1--2 μsec long. The change with time in the phase shift of the 3-centimeter waves passed through the plasma was measured during the period between the pulses.

The magnitude of this shift determines the average concentration of the electrons. The characteristic curves of the concentration change on a semilogarithmic scale are shown in Fig. 20. As can be seen from the figure, at concentrations of 10^9--10^{10} cm^{-3}, the plasma decay is exponential. This means that the nonlinear processes, i.e., the diffusion caused by electron-ion collisions and the recombination, are not essential. Simple estimates indicate that the establishment of a temperature equilibrium between electrons, ions and neutral gas, and the establishment of a diffusional rate of distribution of the charged particles take place during the earlier stages of plasma decay. This is



[мсек = msec.]

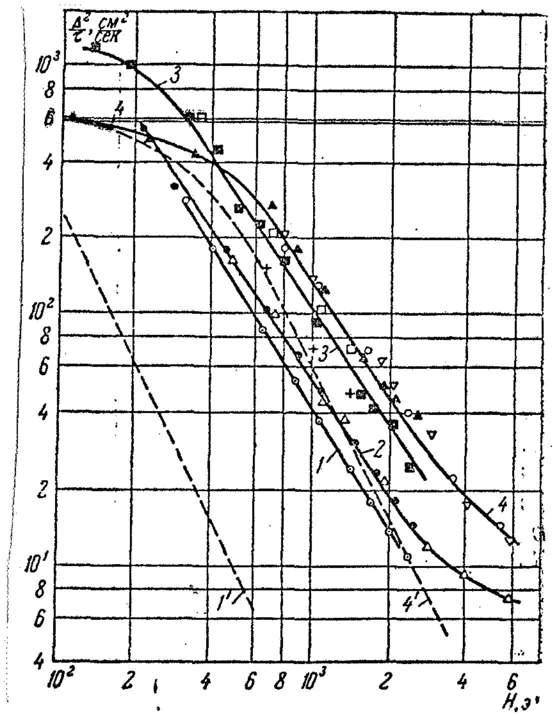
Fig. 20. Variation of the concentration of charged particles in the process of plasma decay.
p = 0.08 mm Hg; 1 - H = 300 Oe; 2 - 450 Oe; 3 - 650 Oe; 4 - 1000 Oe; 5 - 1500 Oe; 6 - 2000 Oe.

confirmed by the identical course of the decay curves at various initial concentrations. Thus, in accordance with (3.41) the "linear" coefficient of transverse diffusion

$$D^0_\perp = \frac{\Lambda_0^2}{\tau_\perp}, \quad \frac{1}{\tau_\perp} = \frac{1}{n}\frac{d\bar{n}}{dt}, \quad \Lambda_0 = \frac{a}{2{,}405}. \qquad (9.1)$$

can be determined from the slope of the curves ln $\bar{n}$ (t) at small concentrations.

Fig. 21 shows the results of a determination of the diffusion the theory. The rate of plasma decay determined experimentally is found to be considerably greater than the theoretical value, particularly at low pressures. The cause of this discrepancy has not been established.* The component of the decay rate which is not dependent on the magnetic field is possibly related to the bulk processes of removal. However,



[сек = sec.]
[э = Oe]

Fig. 21. Dependence of $D_\perp$ on H. Experimental points--curve 1: ⊙-p = 0.025 mm Hg; a = 0.8 cm [95]; curve 2: ●- p = 0.08 mm Hg; a = 0.8 cm [95]; + - p = 0.08 mm Hg; a = 0.7 cm [96]; Δ- p = 0.08 mm Hg; a = 0.5 cm; curve 3: ■- p = 0.35 mm Hg; a = 0.8 cm [95]; □- p = 0.32 mm Hg; a = 0.7 cm [90]; curve 4: ▲- p = o.8 mm Hg; a = 0.8 cm [95]; ▽- p = 1.5 mm Hg; a = 0.5 cm; o- p = 1.5 mm Hg; a = 0.2 cm. Theoretical curves: 1' - p = 0.025 mm Hg; 4' - 0.8 mm Hg.

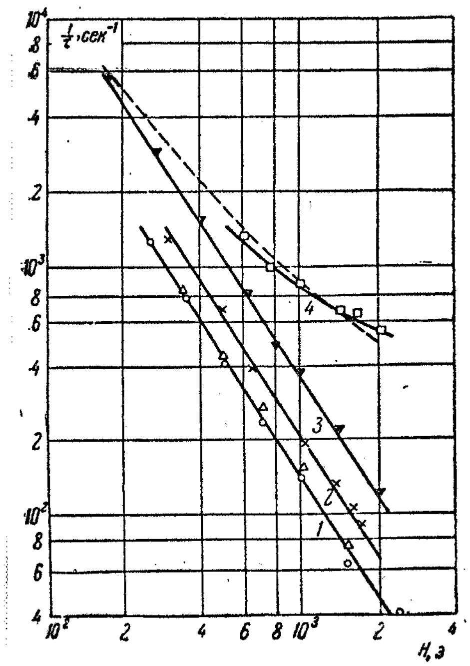
there is an important part of the diffusion coefficient which decreases rapidly with the magnetic field but is weakly dependent on the pressure. It is as if we were dealing with some sort of diffusion mechanism which was not related to the electron-atom collisions. The arbitrary "collision frequency" determining the effectiveness of this mechanism is about 5×10^8 sec^{-1}.

Alikhanov, Demirkhanov and coworkers studied the plasma decay in a glass bottle of large diameter $(2a = 7$ см, $d = 70$ см$)$ [97]. The bottle was completely inserted into a cavity resonator (see Fig. 19c). A

*It is possible that the discrepancy is partly due to the deviation of the electron temperature from the temperature of the gas.

discharge was excited in the gas at one of the resonance frequencies. The concentration at plasma decay was determined from the frequency shift corresponding to the other resonance. The results of measurements of the plasma decay constant in helium at pressures of 0.025--0.2 mm Hg in magnetic fields up to 6,000 Oe are in agreement with the theoretical values. Owing to the comparatively small ratio of the length of the discharge bottle to its diameter $\left(\frac{d}{2a} = 10\right)$ the transverse diffusion of the plasma at magnetic fields greater than 100--500 Oe is masked by the longitudinal diffusion. The data of reference [97] seem to indicate that the rate of transverse diffusion of the plasma in the large-diameter bottle agrees with the theoretical value at magnetic fields less than a few hundred oersteds.

All of the results cited above were obtained in dielectric discharge bottles. The effect of metallic surfaces on the plasma decay was studied in reference [54]. A copper tube 110 cm long and about 2.0 cm in diameter was placed inside a glass discharge bottle. This tube served as the wave guide with which the concentration was measured in the course of the plasma decay. In a homogeneous magnetic field, the rate of decay in the presence of a metallic tube was found to be approximately the same as in a bottle without any metal. To imitate a plasma decay bounded on all sides by a conducting surface, measurements were undertaken in an inhomogeneous field. The length of the solenoid producing the magnetic field was increased so that the ends of the bottle with the metallic tube were outside the solenoid, i.e., so that the lines of force of the magnetic field intersected the metallic surface. As can be seen in Fig. 22, a substantial decrease of the decay constant was observed at large magnetic fields. The replacement of the continuous metallic tube in the bottle



[сек = sec; э = Oe]

Fig. 22. Dependence of $1/\tau$ on H.

Experimental curves: 1, bottle contains a tube made of insulated metallic rings, p = 0.025 mm Hg; O, homogeneous magnetic fields; Δ, ends of the tube outside the field; 2, glass bottle without a metallic tube, p = 0.035 mm Hg; 3, bottle contains a continuous metallic tube, magnetic field is homogeneous, p = 0.08 mm Hg; 4, bottle contains a continuous metallic tube, ends of the tube outside the field, p = 0.08 mm Hg. The broken line represents the calculated curve for the decay in a metallic bottle, p = 0.08 mm Hg.

by a tube consisting of insulated metallic rings (each 10 cm long) leads to the elimination of the observed effects. This shows that the plasma decay constant in the metallic bottle is due to the equalization of potentials at the plasma boundaries, i.e., to the short-circuiting effect. A quantitative comparison of the experimental results with the theory of diffusion in a metallic bottle expounded in 3 of chapter I is difficult, since the causes leading to an accelerated diffusion in a dielectric bottle are not clear. If it is assumed that the effective collision frequency of the electrons in a magnetic field is increased, and if this frequency is determined from experiments on the diffusion in a dielectric bottle, it is possible to calculate the diffusion coefficient in a metallic bottle by means of relations (3.43), (3.45), (1.10) and (1.11). The results of such calculations agree with the experimental data.

III. EXPERIMENTAL INVESTIGATIONS OF THE DIFFUSION OF CHARGED PARTICLES OF A STRONGLY IONIZED GAS IN A MAGNETIC FIELD

10. Decay of Dense Plasma in a Magnetic Field.

At high plasma concentrations, nonlinear processes - the diffusion caused by electron-ion collisions and the recombination - should have a substantial effect on the decay of the plasma. Therefore, by measuring the rate of plasma decay, it is possible to obtain data on the diffusion due to electron-ion collisions.

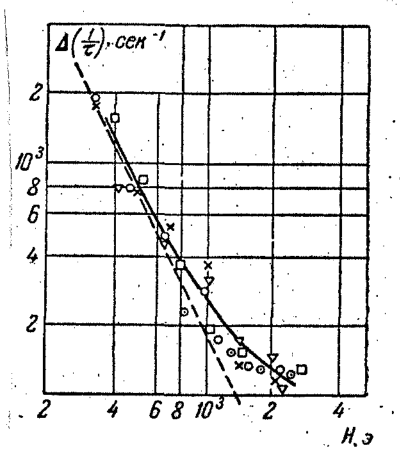
Let us first examine the data obtained in references [94, 54] for measurements of the rate of plasma decay in helium, performed by the waveguide method. In the preceeding section we showed a diagram of these measurements (see Fig. 19a) and the results of investigations at small plasma concentrations when a major part is played by linear processes, and the plasma decay is exponential. At plasma concentrations in excess of 10^9--10^{10} cm^{-3}, the reciprocal of the decay time constant

$$\frac{1}{\tau}=\frac{1}{\bar{n}}\frac{d\bar{n}}{dt} \tag{10.1}$$

increases markedly with the concentration (see Fig. 20). The effectiveness of the nonlinear processes may be characterized by the quantity $\Delta\left(\frac{1}{\tau}\right)=\frac{1}{\tau}-\frac{1}{\tau_0}$ (τ_0 is the decay time constant at small concentrations). An interpretation of the plasma decay curves showed that this quantity changes in approximate proportion to the plasma concentration at magnetic fields of 300--2,000 Oe. Fig. 23 shows the dependence of $\Delta(1/\tau)$ on the magnetic field at a concentration of $\bar{n}=2\cdot 10^{10}\, cm^{-3}$ and at helium pressures of 0.02--0.8 mm Hg. It is evident that the nonlinear increment in the rate of decay at a fixed plasma concentration is almost independent of the pressure in the neutral gas. Analysis of the curve in Fig. 23 may be performed by using relation (3.53):

$$\Delta\left(\frac{1}{\tau}\right)=\frac{1}{\tau}-\frac{1}{\tau_0}=\frac{1}{\tau_{ei}}+\frac{1}{\tau_r}, \quad \frac{1}{\tau_{ei}}\approx\frac{3{,}8\bar{D}_{ei}}{a^2}, \quad \frac{1}{\tau_r}\approx 1{,}2\alpha\bar{n}. \qquad (10.2)$$

At magnetic fields greater than 1,500 Oe, the dependence of $\Delta(1/\tau)$ on the magnetic field is weak. This evidently means that the diffusion is masked by the recombination. Having determined the recombination coefficient ($\alpha=3\cdot10^{-9}$ $см^3$ $сек^{-1}$) from the curve of Fig. 23, one can readily obtain the values of the coefficient $\bar{D}_{ei}$ with the aid of the above formula. The result of a determination of $\bar{D}_{ei}$ at magnetic fields of 500 and 1,000 Oe are given in Fig. 25. According to theory, the quantity $\bar{D}_{ei}$ changes in proportion to the mean concentration (in the range of $5\cdot10^9$—10^{11} $см^{-3}$) and in inverse proportion to the square of the magnetic field strength.



[сек = sec; э = Oe]

Fig. 23. Dependence of $\Delta(1/\tau)$ on H.
Experimental points: ∇ — p= =0,025 мм рт. ст.; × — 0,05 мм рт. ст.; ○ — 0,08 мм рт. ст.; □ — 0,35 мм рт. ст.; ⊙ — 0,8 мм рт. ст.
Theoretical curves: —— $\left(\frac{1}{\tau_{ei}}+\frac{1}{\tau_r}\right)$; – – – $\frac{1}{\tau_{ei}}$
(in the calculations it was assumed that $\alpha=3\cdot10^{-9}$ см³/сек, $T=300°$ K, $\Lambda=8$, $a=0{,}8$ см).

[рт. ст. = Hg]

Anisimov, Vinogradov, Konstantinov and the author [98] used the microwave method of "free space" in studying the plasma decay at high concentrations. In these experiments, the plasma was generated in a cylindrical glass bottle 8.0 cm in diameter filled with helium, by means of pulses of an induction or an electrode discharge. To determine the concentration in the course of the plasma decay, the plasma was sounded at three frequencies simultaneously. By measuring the phases of the reflected waves at several frequencies lower than the "cut-off" frequency, it was possible to determine the positions of the "regions" of reflection for these frequencies (i.e., regions where the concentration is close to critical) [99]. The average concentration was determined from the phase of the wave passing through the plasma.

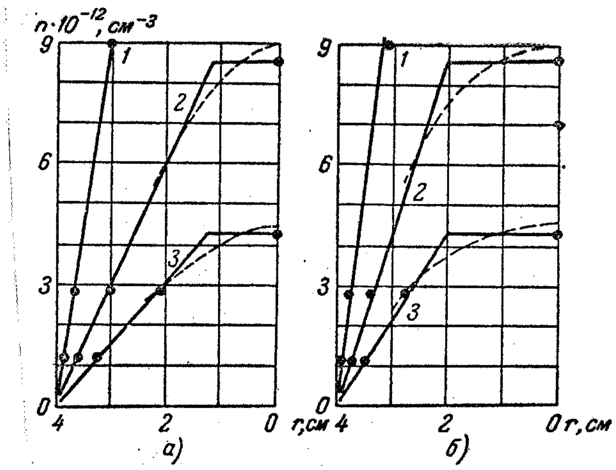


Fig. 24. Radial distribution of charged plasma particles.

а) $H = 0$; 1 — $t = 0,3$ мсек, 2 — 0,5 мсек, 3 — 0,65 мсек;
б) $H = 2000$ э; 1 — $t = 0,4$ мсек, 2 — 0,65 мсек, 3 — 0,9 мсек

(t - time elapsed after the discharge current ceases). The broken lines represent the calculated curves.
[мсек = msec.]

Data were thus obtained on the spatial distribution of the charged particles* (Fig. 24). These data are confirmed by measurements of the glow emitted by the plasma. In the absence of a magnetic field, the concentration distribution is close to the diffusional distribution described by a Bessel function. In a magnetic field of 1,000--2,000 Oe, the concentration distribution is compressed, due to nonlinear processes. This corresponds to the results of the approximate calculation given in 3 of chapter I.

The decay constant of the plasma was determined from the change in the time of the average concentration. At magnetic fields in excess of 1,000--1,500 Oe, the rate of the plasma decay ceases to depend on the magnetic field. The decay curves in this region are in good agreement with the curves obtained from the study of plasma decay in strong

*The trapezoidal distribution form was taken for the treatment of the experimental data.

magnetic fields (H = 30,000 Oe) in the American device the "Stellarator V-1" [100]. As was shown in references [100, 101] the shape of the curves appears to be determined by the recombination caused by collisions of an ion with two electrons.

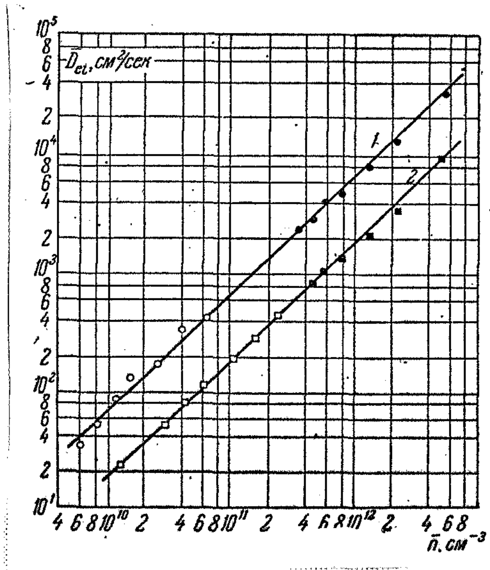
At magnetic fields less than 1,000 Oe, it is possible to determine the "diffusion" time constant by subtracting the rate of recombination from the total rate of decay $\left(\frac{1}{\tau_a} = \frac{1}{\tau} - \frac{1}{\tau_r}\right)$. The dependence of this constant on the magnetic field and the concentration corresponds to the theoretical dependence.

Results of the determination of the coefficient of "nonlinear" diffusion are given in Fig. 25. This figure also shows the results of calculations of $\bar{D}_{ei}$ by means of formula (2.78). In the calculations of the temperature and of the Coulombic logarithm, it was assumed that $T = 300°\ K$, $\Lambda = 8$. Relations (2.80) at plasma concentrations of $5 \cdot 10^9 - 5 \cdot 10^{12}\ cm^{-3}$ lead to values of the Coulombic logarithm of $\Lambda = 2{,}5-7$. It should be noted, however, that since the values of Λ are small and the Debye and Larmor radii are similar under the experimental conditions described, relations (2.80) determine Λ to within several units. For this reason, the agreement between the results of the experimental determination of $\bar{D}_{ei}$ and the theoretical results is satisfactory.

Thus, it follows from references [98, 54] that the nonlinear part of the diffusion coefficient at plasma concentrations of $5 \cdot 10^9 - 5 \cdot 10^{12}\ cm^{-3}$ and magnetic fields up to 1,500 Oe is adequately described by the theory of diffusion based on the consideration of electron-ion collisions.

11. Diffusion in a Stationary Cesium and Potassium Plasma

In gases with a low ionization potential, the surface ionization may be used to generate a plasma. This method of accumulating ions makes it possible to obtain a stationary plasma with a high degree of ionization in the magnetic field. In reference [102], a description is given of a device designed to generate such a plasma; references [103--104] give the results of the diffusion across the magnetic field in this device.



[э = Oe]
[сек = sec]

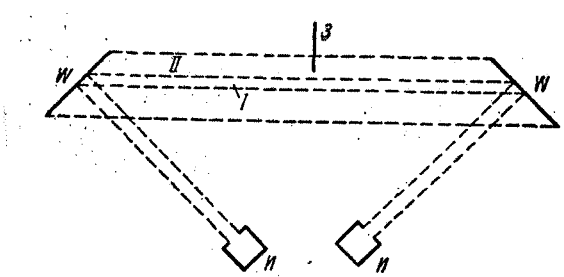
Fig. 25. Dependence of $\bar{D}_{ei}$ on $\bar{n}$.

1—H = 500 э, 2—H = 1000 э, ○, □ Data of ref. /54/; ⊙, ■ Data of ref. /98/; The continuous lines were obtained by calculation, using formula (2.78) (in the calculation it was assumed that T = 3000°K and $\Lambda = 8$).

A diagram of the device is shown in Fig. 26. The source of the charged particles are two tungsten plates W heated to 2,300° K. A beam of cesium or potassium atoms is directed at the central part of these plates from the source S. The ions produced by the surface ionization are retained for a long time in the central portion of the volume (between the plates) by the magnetic field. The heated tungsten plates are also a source of electrons. The amount of the emitted electrons is automatically kept such that the plasma will be quasineutral over its entire volume, with the exception of the layers whose width is of the order of the Debye radius and which are adjacent to the hot plates.

In order to decrease the quantity of neutral atoms in the bulk, the walls of the bottle were kept at a low temperature, about -10°C. The plasma concentration in the central portion of the volume and the degree of ionization are determined by the intensity of the flux of neutral atoms. At a concentration of 10^{10} cm^{-3}, the degree of ionization obtained was about 40%, and at a concentration of 10^{12} cm^{-3}, the degree of ionization was over 99%.

Since the state of the plasma generated in the device described is close to a state of equilibrium, the device is suited for studying diffusion in a magnetic field. Moreover, owing to the high degree of ionization, only the diffusion due to collisions of charged particles with one another, $(\nu_{en} \ll \nu_{ei})$, is essential. To obtain data on the transverse diffusion of the plasma, measurements of the rate of distribution of the concentrations were made by means of a mobile probe of small size (0.025 cm in diameter). The difficulty of these measurements was that the concentration change had to be measured within the confines of the small transverse dimension of the tungsten plates ($\sim$1.4 cm).



3 = P /plate/   И = S /source/

Fig. 26. Diagram of the apparatus for obtaining a stationary plasma.

I--Region of plasma formation; II--Region of transverse diffusion.

It should be noted that the longitudinal distribution of the concentration was uniform, since the charged particles falling on the hot tungsten surface were almost completely reflected from it. Thus, the longitudinal diffusion of the charged particles was absent. As was shown in 3 of chapter I, the characteristic length of the concentration drop, $S_\perp$, is determined by the ratio of the coefficient of transverse diffusion to the recombination coefficient (3.36)

$$s_\perp = \sqrt{\frac{D_{ei}}{\alpha n}} . \quad (11,1) \qquad (11.1)$$

Substituting formula (1.30) for D_{ei} into (11.1), we find

$$s_{\perp}=\frac{c}{eH}\sqrt{\frac{2\nu_{ei}m_eT}{\alpha n}}. \tag{11.2}$$

Since for the conditions of the experiment described the Larmor radius of the electrons is less than the Debye radius $(\bar{\varrho}_e < r_d)$, in calculating the collision frequency V_{ei} the Coulombic logarithm should be determined from formula (2.80b).

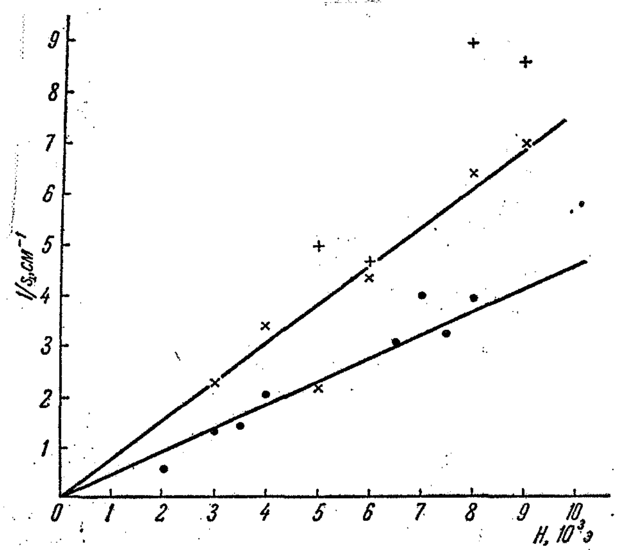


Fig. 27. Dependence of $1/s_{\perp}$ on H.

● — $n = 2{,}5\cdot10^{10}$ см$^{-3}$, × — $3\cdot10^{11}$ см$^{-3}$; + — $5\cdot10^{11}$ см$^{-3}$.

/э = Oe /

The results of the measurements of the characteristic length $1/S_{\perp}$ in cesium are shown in Fig. 27. Similar results were obtained for potassium. It is obvious from the graph that the quantity $1/S_{\perp}$ is approximately proportional to H. Accordingly, the coefficient of transverse diffusion D_{ei} is inversely proportional to the square of the magnetic field strength, as should be expected.

The absolute values of the diffusion coefficient should not be obtained from measurements of $S_{\perp}$ (see (11.1)) since the exact value of the recombination coefficient is not known. If use is made of formula (11.2), obtained with the assumption that the plasma diffusion is caused by electron-ion collisions, the recombination coefficient α may be determined from experimental data. It turns out that the coefficient α grows with the coefficient ($\sim n^{1/2}$), and when $\bar{n} \approx 5 \cdot 10^{11}$ cm^{-2} it is approximately equal to 3×10^{-10} cm^3/sec. This value agrees in order of magnitude with the data of the direct estimation of α and with the results of measurements under other conditions. A certain deviation may be attributed to the inaccuracy of the measurement of the concentration and temperature of the plasma. The dependence of α on n agrees also with the theoretical concept of recombination for collisions of an ion with two electrons. The observed agreement between the data means that the order of magnitude of the coefficient of transverse diffusion D_{ei} is correctly determined by the theoretical formula (2.78).

12. Diffusion in the Presence of a Current Passing Through the Plasma

The behavior of a strongly ionized plasma in a magnetic field has been investigated in many experiments in recent years in connection with the problem of controlled thermonuclear reactions. However, in almost all of these experiments, the lifetime of the charged particles was substantially shorter than the diffusion lifetime. It was determined in the various experiments by the appearance of hydromagnetic plasma instabilities, by the escape of particles along the lines of force of the magnetic field, and by bulk processes (see reference [1]). We shall confine ourselves in this paper to a brief exposition of certain experimental results obtained with toroidal plasma devices with a strong longitudinal magnetic field under conditions where the most dangerous hydromagnetic instabilities produced by the discharge current have been removed.

The most detailed investigations of the lifetime of particles were carried out with the American devices "Stellarator V-1" and "V-3" [105--107]. In these devices, the gas-discharge chamber is a torus "twisted" into a figure eight, so as to compensate for the particle drift. The plasma is generated in an external magnetic field (up to 40 kOe) parallel to the axis of the chamber. In the experiments described, the ionization of the gas and the heating of the plasma were accomplished by a longitudinal electric field (0.05--0.3 v/cm). The highest value of the longitudinal current through the plasma was chosen so that the instabilities due to the helical distortion of the plasma column of lower types (with a small m characterizing the azimuthal symmetry of the perturbation) could not arise, i.e., so that the Kruskal-Shafranov stability condition would be obeyed (see reference [1]).

The plasma concentration was measured during the period of ionization and current heating. The measurements were made by means of the phase shift of the wave which passed through the plasma, using 8-millimeter and 4-millimeter interferometers. The characteristic curves of the change in the plasma parameters in hydrogen are shown in Fig. 28. In the first stage of the process (before the concentration maximum), the gas becomes ionized. At the instant of maximum current (I), the gas in the chamber is almost completely ionized. This is shown by the considerable attenuation of the hydrogen glow (the H_β line). The decrease in the plasma concentration after the maximum is due to the escape of charged particles from the plasma (most of the escaping particles are apparently absorbed on the wall). By analyzing the curves of the concentration change on the basis of reasonable assumptions on the ionization and disassociation of the hydrogen molecule and on the character of the desorption of the gas from the walls, the authors of references [106, 107] determined the time constant of the escape of the particles from the plasma in the ionization stage and in the stage of decrease of the concentration τ. The value of τ was found to be 3-4 orders of magnitude less than the time of the diffusion across the magnetic field caused by collisions of charged particles. In order to find out whether this is due to the instabilities caused by the helical distortions of columns of higher types (with a greater m), experiments were undertaken with additional stabilizing windings of the triple spiral type. The inclusion of stabilizing windings did not produce any substantial change in τ. Measurements showed that τ increased with the magnetic field ($\tau \sim \sqrt{H}$ according to the data of reference [106], $\tau \sim H$ according to those of reference [107]) and decreased with the effective radius of the chamber (according to the data of reference [107] $\tau \sim a^2$). To determine the dependence of τ on the electron temperature, studies were made under conditions of "constant electron temperature." The constancy of the temperature (i.e., of the plasma conductivity) was achieved by a special programming of the voltage source. The magnitude of τ was found to be proportional to T_e. On the basis of measurements of τ in hydrogen at an initial pressure of 10^{-4}–10^{-3} mm Hg, a plasma concentration of 10^{12}--10^{13} cm^{-3}, an electron temperature of 2-20 ev, a magnetic field of 5,000--40,000 Oe and a diameter of the plasma column of 1-4 cm, the authors of reference [107] obtained an empirical formula for the coefficient of "anomalous" diffusion

$$D_{\perp} = \frac{a^2}{5{,}8\tau} \approx 2 \cdot 10^4 \frac{T_e}{H} \ (\mathrm{cm}^2/\mathrm{sec}) \qquad (12.1)$$

(where T_e is in ev and H is in kOe).

Let us note that the results of the various measurements are rather contradictory and for this reason the empirical formula for $D_{\perp}$ is very approximate.

Yavlinskii and coworkers [108] also detected an anomalous diffusion in experiments with the toroidal device "Tokamak-2". For an initial hydrogen pressure of 5×10^{-4}--10^{-3} mm Hg, a plasma concentration of $\sim 10^{13}$ cm^{3}, a longitudinal magnetic field of about 6,000 Oe, a longitudinal electric field of 0.1--0.25 v/cm^{-1}, and a diameter of the plasma column of 20 cm, the lifetime of the charged particles was found to be approximately 600 μsec. This value agrees in order of magnitude with the value given by the empirical formula.

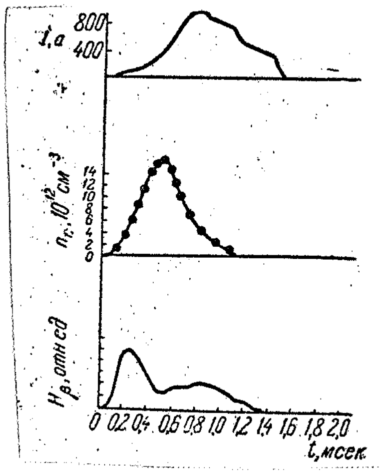


Fig. 28. Curves of the change in plasma parameters in the "Stellarator V-1" device.

[отн сд = rel. un.]
[мсек = msec.]

Investigations were undertaken with the "Stellarator V-1" device with the purpose of finding out at what minimum currents through the plasma the anomalous diffusion arises [109]. The investigations were conducted after the discharge current had ceased during the period of plasma decay, at a magnetic field of 18 kOe. As was shown in ref. [100] the plasma decay is determined under these conditions by the bulk recombination of electrons and ions (see the preceeding section). A longitudinal electric field (frequency 20 MC, amplitude 0.01-0.03 v/cm) was induced in the decaying plasma. Characteristic curves of the variation in the current through the plasma and of the plasma concentration in helium at a pressure of 6×10^{-4} mm Hg are shown in Fig. 29. At first the plasma is heated (as was shown in ref. [100], $T_e \sim 0.1$ ev during the plasma decay). For this reason, the recombination rate decreases, and the plasma conductivity, and hence the current through the plasma, increase. At some critical current, the rate of decay increases sharply*. This rise is attributed in ref. [109] to the appearance of an anomalous diffusion. It has been established that the critical current is approximately proportional to the plasma concentration in the range of 3×10^{11}--3×10^{12} cm^{-3}. The value of the critical current density at a plasma concentration of $\sim 10^{12}$ cm^{-3} and a helium pressure of 2×10^{-4}--4×10^{-3} mm Hg is approximately 0.2--0.3 A/cm^2. Similar results were obtained in argon and hydrogen.

Thus in a strongly ionized plasma, the longitudinal current causes an anomalously rapid escape of particles across the magnetic field, i.e., an anomalous diffusion.

Many studies proposed to explain the anomalous diffusion by the appearance of various plasma instabilities such as the excitation of ionic

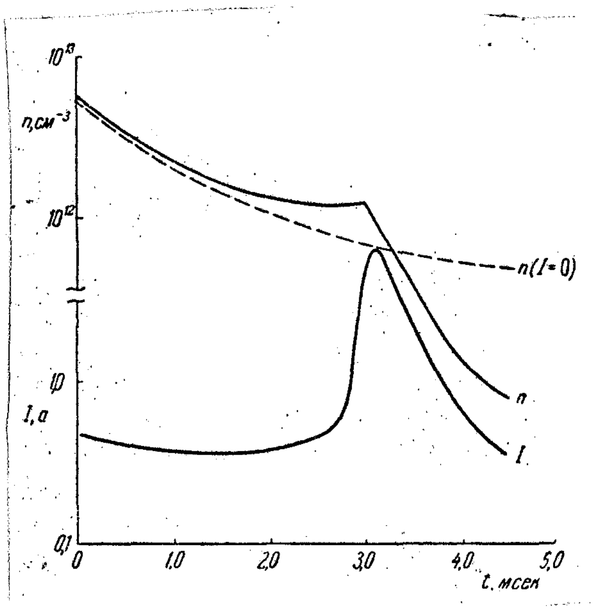


Fig. 29. Curves of the change in the current through the plasma and of the plasma concentration.

[мсек = msec]

*The decrease in the current through the plasma at the end of the decay is probably due to the decrease in the plasma conductivity which at that time is determined by the collisions of electrons with neutral atoms.

oscillations [110, 111] and the appearance of the current-convective instability due to the temperature gradient [112]. At the present time, the evidence is not sufficient for adopting any of the proposed explanations. The nature of the anomalous diffusion thus far remains unclear.

CONCLUSION

Analysis of the results of the experimental investigations has shown that in many cases the diffusion of charged plasma particles across a strong magnetic field occurs much faster than predicted by the theory explaining the diffusion by particle collisions.

The results of experiments on diffusion in the presence of a current through the plasma or a strong directed motion of particles (in the positive column of a discharge, in a discharge with oscillating electrons and a strongly ionized plasma with a current) correspond to the theory but only at moderate magnetic fields. As the magnetic field rises beginning at some critical value, the diffusion rate increases. At the same time, intense plasma oscillations appear which indicate the onset of an instability.

In experiments where the state of the plasma is close to the equilibrium state (decaying plasma in a homogeneous magnetic field, stationary current-free plasma in cesium and potassium), an increase in the magnetic field led to a monotonic decrease of the diffusion rate which was in approximate agreement with the collisions theory $(D_{\perp} \sim 1/H^2)$. However, in a number of experiments on plasma decay, the value of the measured coefficient of transverse diffusion was found to be substantially higher than the expected value at a small collision frequency, as if the effective collision frequencies were greater than the calculated value by about 10^8 sec^{-1}. It is necessary to note that the observed diffusion was never slower than the diffusion caused by collisions with a frequency of 10^7--10^8 sec^{-1}.

The anomalously rapid transport of plasma particles across a magnetic field (anomalous diffusion) observed in the various experiments should naturally be compared with the development of instabilities, whatever they may be, and with the oscillations and noises present in the plasma.

As early as 1948, Bohm suggested the existence of an anomalous mechanism for particle transfer across a magnetic field associated with plasma oscillations* [59] to explain the results of investigations of the diffusion of charged particles from a low-pressure stationary plasma. The plasma oscillations are associated with the appearance of alternating electric fields. The drift of the charged plasma particles caused by these fields (in the plane perpendicular to the magnetic field) leads to a "collisionless" particle diffusion. Without deriving it, Bohm cites the following expression for the coefficient of diffusion across a magnetic field due to oscillations:

$$D_{\perp} = \frac{cT}{16eH}.$$

Although this formula was not justified, it is frequently used in the analysis of experimental results.

To explain the anomalous effects observed in the various experiments, a series of concrete postulations have been proposed concerning the character of the instabilities that arise, some of which we mentioned in the respective sections of this survey.

*As was indicated on p. 61, the results of the measurements of the concentration distribution for diffusion from a stationary plasma, described in Bohm's article [59], may be explained without bringing in the hypothesis of anomalous diffusion.

We have noted that at least in one case it was possible to definitely identify an instability caused by the anomalous diffusion. A helical "current-convective" instability described by Nedospasov and Kadomtsev [81] arises in the positive column of a discharge. The development of this instability at large magnetic fields leads to the appearance of a turbulent plasma state characterized by a broad spectrum of oscillation. The difficulty of treating diffusion under conditions of developed instability is due to the necessity of solving a system of nonlinear equations. Kadomtsev was able to formulate a theory of turbulent diffusion in the positive column of a discharge on the basis of analogy with turbulent motion in fluids [85]. The results of the theory agree with experimental data.

Let us mention two more papers which evaluate the coefficient of anomalous diffusion accompanying the development of instabilities. These papers consider a completely ionized plasma in the presence of the discharge current. In Spitzer's article [111], a determination was made of the coefficient of anomalous diffusion due to the excitation of ion-sonic waves. The expression for the diffusion coefficient is found to be similar to the expression given by Bohm. Kadomtsev obtained a relation for the diffusion coefficient for a developed current-convective instability associated with the temperature gradient in a completely ionized plasma [112]. Thus far, it has not been possible to compare the results of these studies with experimental data.

In some studies attempts were made to find ways of describing the anomalous diffusion which are not related to an analysis of concrete types of instabilities.

In Ecker's paper the increased collision frequency of electrons and ions are formally introduced into the diffusion equations in order to describe the anomalous diffusion [113]. Taylor undertook an investigation with the purpose of establishing limits within which the coefficient of anomalous diffusion should be located [114, 115]. He came to the conclusion that the coefficient of transverse diffusion of ions in a completely ionized gas cannot exceed Bohm's coefficient by more than four times. In the work of Yoshikawa and Rose [116] there is an attempt to establish a relation between the coefficient of diffusion in a turbulent plasma, the intensity of oscillations, and the mean square fluctuation of the concentration, with some rather general assumptions on the character of turbulent motions. We shall not dwell on the above mentioned studies. Let us only observe that these investigations cannot provide an answer to the question concerning the conditions under which the anomalous diffusion arises.

In connection with the problem of the conditions of development of anomalous diffusion, the problem of the so-called universal instability is particularly interesting, i.e., an instability whose development is not associated with the current through the plasma and may be caused by small concentration and temperature gradients. One type of such instability was observed in the theoretical work of Rudakov and Sagdeev [117]. They found that for certain relations between the concentration and the temperature gradient $\left(\frac{\nabla(\ln T)}{\nabla(\ln n)} > 2\right.$ or $\left.\frac{\nabla(\ln T)}{\nabla(\ln n)} < 0\right)$ oblique ion-sonic waves should be excited in the plasma. An examination of the effects associated with the finiteness of the Larmor radius of the ions, carried out by Kadomtsev and Timofeev [120], Galeev, Oraevskii and Sagdeev [121], and Mikhailovskii and Rudakov [122] has shown that this instability may also arise for a more real relationship between the concentration and the temperature gradients (when $\nabla(\ln n) > \nabla(\ln T)$).

The study of the conditions under which a universal plasma instability arises, and the determination of the coefficient of the diffusion accompanying this instability are at the present time some of the most important problems in the experimental and theoretical investigations of transport in a plasma.

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Proofreading note. Listed below are papers dealing with the diffusion of charged plasma particles in a magnetic field, published after the present article was submitted for publication.

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